1 Reasoning with Shapes

Module Pacing: 46 Days

Topic 1: Using a Rectangular Coordinate System

ELPS: 1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Topic Pacing: 17 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**The Squariest Square** From Informal to Formal Geometric Thinking	Through a series of activities, students consider the range of geometric reasoning from informal to formal. To start, students attempt to sketch a "perfect" square and discuss the properties of a square. They analyze a diagram with three squares, create specific angles within the squares, use a protractor to determine their measures, and compare the sum of the measures with their classmates' results. They consider a conjecture about the sum of the measures. To determine whether this conjecture holds true in a second case, they measure the angles in a larger version of the diagram. To move toward generalization, students use patty paper to further analyze the conjecture that the angle measures sum to 90°. The diagram is then expanded through rigid motions to create other geometric properties that students can consider to formally verify the proof, although this final step is not required. They conclude that informal reasoning involves measurements, while formal reasoning involves properties.	• Mathematicians make *conjectures*, test predictions, experiment with patterns, and consider arguments and different perspectives. • Mathematical reasoning can be used to validate a conjecture.	G.4A **G.5A**	1
2	**Hip to Be Square** Constructing on a Coordinate Plane	Students consider how a coordinate plane can be constructed using squares. They start by completing geometric constructions using patty paper or a compass and a straightedge. They analyze Worked Examples to construct perpendicular lines, perpendicular bisectors, and duplicated line segments. Students construct a square and then describe how rigid motions can be applied to create a coordinate plane. They then describe rigid motions that can be used to create two-dimensional shapes on a coordinate plane. Students also relate a sequence of translations to the slope of a line.	• When you *construct* geometric figures, you create exact figures using only a *compass* and *straightedge* or patty paper. • The *midpoint* of a segment is a *point* that divides the segment into two congruent segments. • A *segment bisector* is a *line*, *line segment*, or ray that divides a line segment into two line segments of equal length. • A *perpendicular bisector* is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment. • Any point on a perpendicular bisector is equidistant to the endpoints of the original segment it bisects. • The *diagonals* of a square are congruent, bisect each other, are perpendicular to one another, and bisect the angles of the square. • A coordinate plane can be created by constructing a square and applying *rigid motion* transformations to the square.	G.3C G.5B G.5C	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Ts and Train Tracks** Parallel and Perpendicular Lines	Students investigate segments on a coordinate grid and use patty paper to create parallel and perpendicular segments. They then construct parallel lines off the coordinate plane and graph parallel lines on the coordinate plane. Students identify perpendicular lines on the coordinate plane, use a rigid motion transformation to demonstrate that their slopes are negative reciprocals, and extend their understanding of perpendicular lines to include horizontal and vertical lines. They provide an explanation to demonstrate that if two lines are parallel, then their slopes are equal.	• The 90° rotation of a line creates a line perpendicular to the original line. • Perpendicular lines have slopes that are negative reciprocals of each other. • The translation of a line creates an identical line or a line parallel to the original line. • Parallel lines have equal slopes.	**G.2C** **G.5A** G.5B G.5C	**2**
4	**Where Has Polly Gone?** Classifying Shapes on the Coordinate Plane	Students use a Venn diagram to sort quadrilaterals and triangles based on shared properties. They are introduced to the Distance Formula and use it to calculate the lengths of sides of triangles and quadrilaterals on the coordinate plane. Students also use the slope formula to determine whether opposite sides of a quadrilateral are parallel and whether consecutive sides of a quadrilateral are perpendicular. They use these skills to classify triangles and quadrilaterals that lie on a coordinate plane or to determine the fourth point of a quadrilateral when given three points. Students are then introduced to the Midpoint Formula and use it to classify secondary figures formed when connecting the midpoints of consecutive sides of quadrilaterals. Finally, students consider translations as a strategy to identify the coordinates that create quadrilaterals with parallel sides.	• The *Distance Formula* states that the distance d between points (x_1, y_1) and (x_2, y_2) on a coordinate plane is given by the equation $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. • The Distance Formula can be used to classify triangles and quadrilaterals based on side lengths. • The slope formula can be used to determine whether opposite sides are parallel or consecutive sides are perpendicular in a quadrilateral on the coordinate plane. • The *Midpoint Formula* states that if (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. • The use of translations is an efficient strategy when determining endpoints of parallel segments on a coordinate plane.	**G.2B** **G.9B**	3

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**In and Out and All About** Area and Perimeter on the Coordinate Plane	Students calculate the perimeter and area of rectangles and triangles on the coordinate plane. They double dimensions of figures and explain how this affects the area of the figure; they also translate figures on the coordinate plane to more efficiently determine their perimeter and area. Students algebraically determine the non-vertical height of a triangle as they treat each side as the base; they then use the height to calculate the area of the triangle. They conclude that the area of a triangle remains the same regardless of the side considered as the base and the height determined by that base. Next, students divide a composite figure into various known polygons to compute its area. They then consider real-world situations requiring them to calculate the perimeter and area of polygons that lie on a coordinate plane using the Distance Formula and decomposing the polygons into triangles and rectangles. As students determine distances represented as the area under the curve of velocity-time graphs, they investigate how proportional and non-proportional changes in the linear dimensions of a shape affect its perimeter and area. Students develop a strategy for calculating areas of regular polygons.	• Rigid motion transformations (translations, rotations, and reflections) can be used to change the position of figures on the coordinate plane. • Performing translations on figures can help to compute perimeter and area more efficiently. • Non-vertical heights of a figure can be calculated algebraically using formulas, writing equations, and solving a system of equations. • The area of a triangle is the same regardless of what base and height of the triangle are used in the calculation. • A *composite figure* is a figure that is formed by combining different shapes. • Polygons can be divided into a combination of triangles and rectangles to help determine their area. • The area of a composite figure is determined by dividing the figure into familiar shapes and using the area formulas associated with those shapes. • The Distance Formula, slope formula, and the Pythagorean Theorem can be used to determine the area of polygons and composite figures on the coordinate plane. • A velocity-time graph can model acceleration, and distance can be determined by calculating the area under a curve. • When the dimensions of a plane figure change proportionally by a factor of k, its perimeter changes by a factor of k, and its area changes by a factor of k_2. • When the dimensions of a plane figure change non-proportionally, its perimeter and area increase or decrease non-proportionally. • A regular polygon is a polygon with congruent sides and congruent angles. • A regular n-gon can be decomposed into n-congruent triangles. • The area of a regular n-gon can be calculated by determining the area of one of the n congruent triangles and multiplying by n.	**G.2B** **G.2C** G.3C **G.10B** G.11A **G.11B**	4
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					4

Topic 2: Rigid Motions on a Plane

ELPS: 1.A, 1.C, 1.D, 1.E, 1.F, 1.G, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.D, 4.F, 4.J, 4.K, 5.B, 5.E, 5.F, 5.G

Topic Pacing: 18 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Put Your Input In, Take Your Output Out** Geometric Components of Rigid Motions	Students develop the concept that geometric rigid motion transformations can be considered as functions, with rotations, reflections, and translations as the operations. Translations can be described using lines and line segments. Reflections can be described using lines. Rotations can be described using rotation angles. The inputs and outputs are geometric shapes. Each input and its corresponding output have the same size and shape.	• Pre-images transformed by rigid motions such as translations, reflections, and rotations are congruent to their images. • Translations of lines produce parallel lines. • Points and lines are essential building blocks of geometry and of geometric transformations. • A line is a geometric object such that if any part of the line is translated to another part of the line so that the two parts have two points in common, then the first part will lie exactly on top of the second part. • Translations can be described using lines and line segments. Reflections can be described using lines. Rotations can be described using *rotation angles*.	**G.3B** G.3C	**1**
2	**Bow Thai** Translations as Functions	Students analyze transformation machines and conclude that translations along parallel lines always produce images that are congruent to their pre-image, while translations along rays with a common endpoint produce dilations or images that are similar to, but not congruent to, their pre-image. The term *isometry* is defined to label these differences, with the understanding that any rigid motion transformation that preserves size and shape is an isometry. Students then engage in a context involving an animated website where they learn and use function notation to represent geometric translations.	• Translations along parallel lines are rigid motions and always produce images that are congruent to the pre-image. • A *translation* is a function, represented as $T_{ab}(P) = P'$, which takes as its input the location of a point P and translates it a distance AB in the direction AB. • *Isometries* are rigid motion transformations that preserve size and shape.	**G.3B** G.3C G.6C	2
		Mid-Topic Assessment			**0**
3	**Staring Back at Me** Reflections as Functions	Students analyze reflections as isometries. They construct a perpendicular bisector of a segment and then conclude that the perpendicular bisector is the line of reflection between the endpoints of the segment. Students investigate reflections as functions using the context from the previous lesson, use function notation to represent geometric reflections, and construct lines of reflection. They combine what they learned in this lesson and the previous lesson to identify sequences of translations and reflections to demonstrate that two figures are congruent.	• The perpendicular bisector of a line segment is a line of reflection between the two endpoints of the segment. • Reflections are isometries. • A *reflection* is a function, R_{ℓ}, which takes as its input, P, the location of a point with respect to some line of reflection, ℓ, and outputs $R_{\ell}(P)$, or the opposite of the location of P with respect to the line of reflection. • The *Perpendicular Bisector Theorem* states: "If two points are equidistant from a third point, the third point lies on the perpendicular bisector of the segment connecting the two points."	**G.3B** G.3C G.5B **G.6A** G.6C	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
4	**Turn Yourself Around** Rotations as Functions	Students analyze rotations. First they use concentric circles to rotate a triangle and determine that rotations are isometries. They are then introduced to the notation for the rotation function and use it to rotate any figure using only a protractor and ruler. Students then reverse the process and identify the center of rotation and angle of rotation given a pre-image and image of a figure. As in the previous lesson, they identify sequences of transformations to demonstrate that two figures are congruent. Students then use a graphic organizer to summarize what they have learned about translation, reflection, and rotation isometries.	• Rotations are isometries. • A *rotation* is a function, $R_{E,t}(P) = P'$ that maps its input, a point P, to another location, P'. This movement to a new location is defined by a center of rotation, E, and a rotation angle, t. • The center of rotation lies on the perpendicular bisector of each pair of corresponding points of a pre-image and its rotated image. For this reason, the center of rotation is the point of intersection of any two of these perpendicular bisectors.	**G.3B** G.3C G.5B G.6C	3
5	**Slide, Flip, Turn: The Latest Dance Craze?** Translations, Rotations, and Reflections on the Coordinate Plane	Students recall what they know about transformations of functions by examining the graph of the basic function, $f(x)$, and its transformed graph $g(x)$. Students then cut out a model of a trapezoid and translate, rotate, and reflect the model on a coordinate plane to determine how transformations affect the coordinates of the figure. Compositions of transformations are explored on the coordinate plane.	• When a horizontal translation occurs on a coordinate plane, the x-coordinates of the pre-image change, but the y-coordinates remain the same. • When a vertical translation occurs on a coordinate plane, the y-coordinates of the pre-image change, but the x-coordinates remain the same. • When a point or image on a coordinate plane is rotated 90° counterclockwise about the origin, its original coordinates (x, y) change to $(-y, x)$. • When a point or image on a coordinate plane is rotated 180° counterclockwise about the origin, the original coordinates (x, y) change to $(-x, -y)$. • When a point or image on a coordinate plane is rotated 270° counterclockwise about the origin, the original coordinates (x, y) change to $(y, -x)$. • When a point or image on a coordinate plane is rotated 360° counterclockwise about the origin, the original coordinates (x, y) do not change. • When a point or image on a coordinate plane is reflected over the x-axis, the original coordinates (x, y) change to $(x, -y)$. • When a point or image on a coordinate plane is reflected over the y-axis, the original coordinates (x, y) change to $(-x, y)$.	G.3A **G.3B** G.3C	3

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
6	**OKEECHOBEE** Reflectional and Rotational Symmetry	Students explore reflectional and rotational symmetry within a figure using patty paper prior to formal definitions being provided. They then analyze these symmetries in more depth as they relate the number of lines of symmetry and the measures of angles of rotation to specific types of figures. Students identify reflectional and rotational symmetry in letters of the alphabet and some titles in this lesson. They also identify the relationship between the rotational symmetries of a regular figure and the measure of each of its interior angles.	• A plane figure has *reflectional symmetry* if you can draw a line so that the figure to one side of the line is a reflection of the figure on the other side. • A plane figure has *rotational symmetry* if you can rotate the figure more than 0° and less than 360° and the resulting figure is the same as the original figure. • An individual figure may have horizontal symmetry, vertical symmetry, and/or rotational symmetry. • A regular polygon of n-sides has n lines of symmetry. • The measure of the angle of rotation of a regular polygon with n sides is $\frac{360°}{n}$, which is the supplement of the measure of each of its interior angles.	G.3D	1
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					5

Topic 3: Congruence Through Transformations

ELPS: 1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Topic Pacing: 11 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Elemental** Formal Reasoning in Euclidean Geometry	Students are introduced to formal reasoning as a foundation for proving geometric theorems. They begin by writing a counterexample to demonstrate a statement is false. Students then analyze conditional statements, determine truth values for all possible cases, and summarize the results in a truth table. Euclidean geometry is introduced as a system built by postulates and proven theorems, and students analyze the Linear Pair Postulate, Segment Addition Postulate, and Angle Addition Postulate.	• The two reasons why a conclusion may be false is either the assumed information is false or the conclusion does not follow from the hypothesis. • A *counterexample* is used to show a general statement is not true. • A conditional statement is a statement that can be written in the form "If *p*, then *q*." The variable *p* represents the hypothesis and the variable *q* represents the conclusion. • A *truth value* is whether or not a conditional statement is true or false; it is true if the conditional statement could be true, and it is false if the conditional statement could not be true. • *Truth tables* are used to organize truth values of conditional statements. • A postulate is a statement that is accepted without proof. A theorem is a statement that can be proven. • The *Linear Pair Postulate* states: "If two angles form a linear pair, then the angles are supplementary." • The *Segment Addition Postulate* states: "If point *B* is on $\overline{AC}$ and between points *A* and *C*, then $AB + BC = AC$." • The *Angle Addition Postulate* states: "If point *D* lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$." • There are multiple systems of geometry and non-Euclidean geometry including *spherical geometry*. • One example of non-Euclidean geometry is spherical geometry.	G.4A G.4B **G.4C** G.4D	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
2	**ASA, SAS, and SSS** Proving Triangle Congruence Theorems	Students use what they have learned in the previous topic: (1) isometries preserve distances and angle measures, (2) any point in the plane can be reflected across a line to map to another point in the plane, and (3) a point is equidistant from two other points if and only if it lies on their perpendicular bisector. They use these facts to create and verify proofs of the SSS, SAS, and ASA Congruence Theorems using rigid motion transformations. Students then explore some non-examples of congruence theorems (AAA and SSA).	• A proof is a series of statements and corresponding reasons forming a valid argument that starts with a hypothesis and arrives at a conclusion. • The *Side-Side-Side (SSS) Congruence Theorem* states: "If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent." • *Corresponding parts of congruent triangles are congruent*, abbreviated as CPCTC, is often used as a reason for stating congruences in geometric proofs after triangles have been proven congruent. • The *Side-Angle-Side (SAS) Congruence Theorem* states: "If two sides and the *included angle* of one triangle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent." • The *Angle-Side-Angle (ASA) Congruence Theorem* states: "If two angles and the *included side* of one triangle are congruent to the corresponding angles and included side of another triangle, then the triangles are congruent." • Triangle congruence theorems such as SSS, SAS, and ASA can be proven using rigid motion transformations.	**G.6B** G.6C	3
3	**I Never Forget a Face** Using Triangle Congruence to Solve Problems	Students determine whether triangles are congruent using SSS, SAS, and ASA. First, they explain how a triangle congruence theorem can be applied to a real-world situation. They then determine whether triangles in complex diagrams are congruent. Students use the coordinate plane to assist in measurements or transformations to determine whether triangles are congruent. Finally, they apply transformations to create an original wallpaper design.	• The SSS, SAS, and ASA Congruence Theorems can be applied to solve real-world and mathematical problems. • Congruent parts of triangles can be depicted from a diagram rather than stated. These can be instances where two triangles share a common side or angle. • The SSS, SAS, and ASA Congruence Theorems can be applied to triangles on or off the coordinate plane.	**G.2B** **G.6B** G.6C	**2**
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					3

2 Establishing Congruence

Module Pacing: 47 Days

Topic 1: Composing and Decomposing Shapes

ELPS: 1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.F, 4.K, 5.E

Topic Pacing: 18 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Running Circles Around Geometry** Using Circles to Make Conjectures	Students explore and identify lines and angles associated with the interior and exterior of a circle. Circles are used to make conjectures about line and angle relationships that students will prove throughout the course. They begin by trying to draw perfect circles freehand and exploring criteria to judge the quality of the circles. Students then construct a circle, a perpendicular bisector to the diameter, and a chord to identify circle parts; conjecture about points on a perpendicular bisector; and create a special right triangle. Next, they construct a parallel line through the circle's center to conjecture about angle relationships given parallel lines that are intersected by a transversal. Students make conjectures related to inscribed angles and angles formed at the point of tangency when two lines intersect at a point outside the circle. Finally, they summarize the lesson by drawing examples of conjectures that were made in the activity.	• When you conjecture, you use what you know through experience and reasoning to presume that something is true. The statement of a conjecture, once proven, is then called a theorem. • Circles can be helpful in constructing geometric figures in order to make conjectures about line and angle relationships.	G.4A **G.5A** G.5B G.5C G.12A	2
2	**The Quad Squad** Conjectures About Quadrilaterals	Students investigate the properties of quadrilaterals and use them to make conjectures. They explore the diagonals of both convex and concave quadrilaterals. Students construct several quadrilaterals from the diameters of concentric circles. They use patty paper to first draw their diagonals and then connect the endpoints of those diagonals to form the sides of each figure. Students use measuring tools to determine the side lengths and interior angle measures. Using this information, they are able to name the quadrilaterals. Students make conjectures about the diagonals and relationships in kites and isosceles trapezoids. They complete a table identifying quadrilaterals with given properties, and then describe how to construct various quadrilaterals given only one diagonal. The term *midsegment* is defined, and students investigate the figure formed by adjacent midsegments of quadrilaterals. They make a conjecture about the measure of the midsegment of a trapezoid in relation to its bases. The term *cyclic quadrilateral* is defined, and students investigate the sum of the measures of opposite angles of different cyclic quadrilaterals to make a conjecture.	• The diagonals of any convex quadrilateral create two pairs of vertical angles and four linear pairs of angles. • Parallelograms, rhombi, and *kites* have diagonals that are not congruent. • Rectangles, squares, and *isosceles trapezoids* have congruent diagonals. • Circles can be helpful in understanding that the diagonals of parallelograms bisect each other, the diagonals of rectangles are congruent, and the diagonals of kites are perpendicular. • The measure and relationship of the diagonals of quadrilaterals can be used to make conjectures about quadrilaterals. • The relationship of the interior angles of quadrilaterals can be used to make conjectures about quadrilaterals. • The *midsegment* of a quadrilateral is any line segment that connects two midpoints of the sides of the quadrilateral. • A quadrilateral whose vertices all lie on a single circle is a *cyclic quadrilateral*.	**G.5A** G.5B G.5C G.6E	3

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Into the Ring** Constructing an Inscribed Regular Polygon	Students learn how to construct three regular polygons: regular hexagons, squares, and equilateral triangles. They start by constructing a regular hexagon inscribed in a circle using two different methods. First, they duplicate 60° angles to create six equilateral triangles sharing the center of the circle as a vertex. Secondly, they construct six adjacent congruent chords the same length as the radius of the circle around the circumference of the circle to inscribe a regular hexagon. Students then construct a square inscribed in a circle using two different methods. First, they use patty paper to rotate a right triangle to create four congruent right triangles sharing the center of the circle as the vertex of the right angle. Secondly, they construct the diagonals of a square by constructing perpendicular diameters of a circle and then connect the endpoints. Finally, students analyze Worked Examples that demonstrate how to bisect an angle using patty paper and a compass and straightedge. Students construct both an equilateral triangle given a side length and an equilateral triangle inscribed in a circle. They demonstrate their construction skills by constructing inscribed angles, a 75° angle, and a regular octagon inscribed in a circle.	• Constructions can be used to duplicate a given angle. • A 60° angle can be constructed by creating an equilateral triangle within a circle. • A regular hexagon can be inscribed in a circle by duplicating 60° angles to create six equilateral triangles sharing the center of the circle as a vertex. • A regular hexagon can be inscribed in a circle by constructing six adjacent congruent chords the same length as the radius of the circle around the circumference of the circle. • When a square is inscribed in a circle, a segment that is a diagonal of the square is also a diameter of the circle. • An *angle bisector* is a line, segment, or ray that is drawn through the vertex of an angle and divides the angle into two congruent angles. Angle bisectors can be constructed using patty paper or a compass and straightedge. • Both an equilateral triangle with a given side length and an equilateral triangle inscribed in a circle can be created using construction tools. • The central angle of a circle is twice the measure of an inscribed angle that intercepts the same arc of the circle. • Constructions can be used to verify geometric theorems.	G.5B G.5C	3

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
4	**Tri- Tri- Tri- and Separate Them** Conjectures About Triangles	Students begin by decomposing the quadrilaterals they investigated in the previous lesson to form the triangles that they investigate in this lesson. They learn how to write the converse of a conditional statement and then explore the converse of the base angles conjecture for isosceles triangles. Students construct an equilateral triangle using two circles and use the diagram to conjecture about the sum of the interior angles of a triangle and the exterior angles of a triangle. Students use a circle diagram to make conjectures about triangle inequality and triangle midsegments. Finally, they summarize the lesson by drawing examples of conjectures that were made in the activities.	• Circles can be helpful in constructing geometric figures to make conjectures about triangles. • A convex quadrilateral can be divided by any one of its diagonals into two triangles. • The *converse* of a statement is different from the original statement and is formed by interchanging the *hypothesis* and *conclusion* of the original statement. • The truth value of a conditional statement and its converse are not necessarily the same. • The base angles of an isosceles triangle are congruent. • A point that lies on a perpendicular bisector of a line segment is equidistant from the endpoints of the line segment. • The measure of the *exterior angle* of a triangle is equal to the sum of the measures of the two remote interior angles. • The sum of the measures of the interior angles of a triangle is 180°. • The length of the third side of a triangle cannot be equal to or greater than the sum of the measures of the other two sides. • The midsegment of a triangle is one-half the measure and parallel to the third side. • A conjecture is a statement believed to be true based on observations. A conjecture must be proved with definitions and theorems to be fully accepted.	G.4A G.4B **G.5A** G.5B G.5C G.5D **G.9B**	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**What's the Point?** Points of Concurrency	Students construct the four points of concurrency for triangles—the incenter, circumcenter, centroid, and orthocenter. They construct these points in acute, obtuse, right, and equilateral triangles. Students construct perpendicular bisectors, angle bisectors, medians, and altitudes to locate the points of concurrency. They use the circumcenter to circumscribe a circle about a triangle and the incenter to inscribe a circle in a triangle. Students use their constructions to make conjectures. Students may use whichever construction tools they are most comfortable with—compass and straightedge or patty paper.	• A *point of concurrency* is a point at which three lines, rays, or line segments intersect. • The *circumcenter* is the point of concurrency of the three perpendicular bisectors of the sides of a triangle, and it is equidistant from each vertex of the triangle. • The circumcenter can be used to circumscribe a circle about a triangle. • The *incenter* is the point at which the three angle bisectors of a triangle are concurrent and it is equidistant from each side of the triangle. • The incenter can be used to construct a circle inscribed in a triangle. • The *median* of a triangle is a line segment formed by connecting a vertex of a triangle to the midpoint of the opposite side of the triangle. • The *centroid* is the point at which the three medians of a triangle are concurrent. • The distance from the centroid to the vertex is twice the distance from the centroid to the midpoint of the opposite side. • The *orthocenter* is the point at which the three altitudes of a triangle are concurrent.	G.5B G.5C	2
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					5

Topic 2: Justifying Line and Angle Relationships

ELPS: 1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.F, 4.K, 5.E

Topic Pacing: 20 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Proof Positive** Forms of Proof	Students apply real number properties to angle measures, line segments, and distances. Postulates and properties are reviewed. Equality statements are distinguished from congruence statements. Proof by construction, flowchart proof, two-column proof, and paragraph proof are introduced. Students identify the given and prove statements in theorems, then use different forms of proof to complete flow chart proofs, two-column proofs, and a paragraph proof. Students prove the Congruent Supplement Theorem and Vertical Angle Theorem.	• The *Addition Property of Equality*, the *Subtraction Property of Equality*, the *Reflexive Property*, the *Substitution Property*, and the *Transitive Property* can be applied to angle measures, segment measures, and distances. • A *construction proof*, *two-column proof*, *flowchart proof*, and *paragraph proof* are all acceptable forms of reasoning about geometric relationships. • The *Right Angle Congruence Postulate* states: "All right angles are congruent." • The *Congruent Supplement Theorem* states: "If two angles are supplements of the same angle or of congruent angles, then the angles are congruent." • The *Vertical Angle Theorem* states: "Vertical angles are congruent."	G.4A G.5B **G.6A**	3

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
2	**A Parallel Universe** Proving Parallel Line Theorems	Students prove the angle relationships when two parallel lines are cut by a transversal. They address the relationships between pairs of corresponding angles, same-side interior angles, alternate interior angles, same-side exterior angles, and alternate exterior angles. Students prove the Corresponding Angles Theorem using translations and the remaining theorems using a flowchart proof, two-column proof, or paragraph proof. After proof plans are described and modeled, students use this method to organize a proof strategy and reasoning path. They also prove the Same-Side Interior Angles Theorem, Alternate Interior Angles Theorem, Parallel Line Converse Theorems, and Perpendicular/Parallel Line Theorem.	• The *Corresponding Angle Theorem* states: "If two parallel lines are cut by a transversal, then corresponding angles are congruent." • The *Corresponding Angle Converse Theorem* states: "If two lines cut by a transversal form congruent corresponding angles, then the lines are parallel." • The *Same-Side Interior Angle Theorem* states: "If two parallel lines are cut by a transversal, then same-side interior angles are supplementary." • The *Same-Side Interior Angle Converse Theorem* states: "If two lines cut by a transversal form supplementary same-side interior angles, then the lines are parallel." • The *Alternate Interior Angle Theorem* states: "If two parallel lines are cut by a transversal, then alternate interior angles are congruent." • The *Alternate Interior Angle Converse Theorem* states: "If two lines cut by a transversal form congruent alternate interior angles, then the lines are parallel." • The *Same-Side Exterior Angle Theorem* states: "If two parallel lines are cut by a transversal, then same-side exterior angles are supplementary." • The *Same-Side Exterior Angle Converse Theorem* states: "If two lines cut by a transversal form supplementary same-side exterior angles, then the lines are parallel." • The *Alternate Exterior Angle Theorem* states: "If two parallel lines are cut by a transversal, then alternate exterior angles are congruent." • The *Alternate Exterior Angle Converse Theorem* states: "If two lines cut by a transversal form congruent alternate exterior angles, then the lines are parallel." • The *Perpendicular/Parallel Line Theorem* states: "If two lines are perpendicular to the same line, then the two lines are parallel to each other."	**G.3B** G.4A G.4B **G.6A**	**2**
Mid-Topic Assessment					1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Ins and Outs** Interior and Exterior Angles of Polygons	Students model the Triangle Sum Theorem by cutting off the angles of a triangle and aligning them to form a line. They then write a proof plan and use a paragraph proof to prove the Triangle Sum Theorem. The Exterior Angle Theorem is given and used to solve problems. Students draw diagonals in a polygon from one vertex to divide the polygon into triangles and look for patterns as the number of sides in the polygon increases. They derive the formulas that determine the sum of the measures of the interior angles of a polygon, the measure of each interior angle of a regular polygon, the sum of the measures of the exterior angles of a polygon, and the measure of each exterior angle of a regular polygon.	• The *Triangle Sum Theorem* states: "The sum of the measures of the interior angles of a triangle is equal to 180°." • The *Exterior Angle Theorem* states: "The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles." • The sum of the measures of the interior angles of a quadrilateral is equal to 360°. • For a polygon with n sides, the sum of its interior angle measures is equal to $180(n - 2)°$. • For a regular polygon with n sides, the measure of each interior angle is equal to $\frac{180(n-2)°}{n}$. • For a polygon with n sides, the sum of the measures of the exterior angles is equal to 360°.	**G.5A** G.6D	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
4	**Identical Twins** Perpendicular Bisector and Isosceles Triangle Theorems	Students use their knowledge of Side-Angle-Side (SAS), Side-Side-Side (SSS), and Angle-Side-Angle (ASA) Congruence Theorems to explain why pairs of triangles are congruent. The term *CPCTC* (corresponding parts of congruent triangles are congruent) is defined as a reason that can be used after two triangles are proved congruent. Students prove the Perpendicular Bisector Theorem using CPCTC and analyze a Worked Example of its converse. Then, students use theorems to demonstrate why the Hypotenuse-Angle (HA) Congruence Theorem and the Angle-Angle-Side Congruence Theorem are valid. Next, students use CPCTC and the construction of auxiliary lines to prove the Isosceles Triangle Base Angles Theorem and the Isosceles Triangle Base Angles Converse Theorem. Next, they analyze a diagram of a constructed equilateral triangle with a perpendicular bisector constructed through a side. They use the diagram to demonstrate algebraically the 30°–60°–90° Triangle Theorem. Students then analyze a diagram of an isosceles right triangle constructed with its right angle at the center of a circle along with the perpendicular bisector of the diameter of the circle. They use the diagram to demonstrate algebraically the 45°–45°–90° Triangle Theorem. Students review the procedure of extracting the square root and the standard math convention of rationalizing the denominator.	• The Perpendicular Bisector Theorem states: “The points on a perpendicular bisector of a line segment are equidistant from the segment’s endpoints.” • The *Perpendicular Bisector Converse Theorem* states: “If a point is equidistant from the endpoints of a line segment, then the point lies on the perpendicular bisector of the line segment.” • The *Isosceles Triangle Base Angles Theorem* states: “If two sides of a triangle are congruent, then the angles opposite these sides are congruent.” • The *Isosceles Triangle Base Angles Converse* Theorem states: “If two angles of a triangle are congruent, then the sides opposite these angles are congruent.” • The *30°–60°–90° Triangle Theorem* states: “The length of the hypotenuse in a 30°–60°–90° triangle is 2 times the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.” • The *45°–45°–90° Triangle Theorem* states: “The length of the hypotenuse in a 45°–45°–90° triangle is $\sqrt{2}$ times the length of a leg.” • The *Hypotenuse-Angle (HA) Congruence Theorem* states: “If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the two triangles are congruent.” • The *Angle-Angle-Side (AAS) Congruence Theorem* states: “If two angles and the non-included side of one triangle are congruent to two angles and the non-included side of another triangle, then the two triangles are congruent.”	G.4B **G.6A** **G.6B** G.6D **G.9B**	3

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**Corners in a Round Room** Angle Relationships Inside and Outside Circles	Students reason about arc measures associated with a clock face and conclude that the measures of two central angles of the same circle (or congruent circles) have corresponding congruent minor arcs. They use a two-column proof to prove one case of the Inscribed Angle Theorem and algebraic reasoning to prove the other two cases. Students then prove two theorems associated with inscribed polygons using the Inscribed Angle Theorem. They explore and prove theorems for determining the measures of angles located on the inside and outside of a circle. They construct a tangent line to a circle from a point outside the circle. A proof by contradiction is provided to show a perpendicular relationship exists when the radius of a circle is drawn to a point of tangency. Finally, students use the theorems they have proved to determine the measures of arcs and angles of a circle.	• The *Arc Addition Postulate* states: "The measure of an arc formed by two *adjacent arcs* is the sum of the measures of the two arcs." • The measure of a central angle is equal to the measure of its intercepted arc. • The *Inscribed Angle Theorem* states: "The measure of an inscribed angle is equal to half the measure of its intercepted arc." • The *Inscribed Right Triangle–Diameter Theorem* states: "When a triangle is inscribed in a circle such that one side of the triangle is a diameter, the triangle is a right triangle." • The *Inscribed Quadrilateral–Opposite Angles Theorem* states: "When a quadrilateral is inscribed in a circle, the opposite angles are supplementary." • The *Interior Angles of a Circle Theorem* states: "If an angle is formed by two intersecting chords or secants of a circle such that the vertex of the angle is in the interior of the circle, then the measure of the angle is half of the sum of the measures of the arcs intercepted by the angle and its vertical angle." • The *Exterior Angles of a Circle Theorem* states: "If an angle is formed by two intersecting chords or secants of a circle such that the vertex of the angle is in the exterior of the circle, then the measure of the angle is half of the difference of the measures of the arcs intercepted by the angle." • The *Tangent to a Circle Theorem* states: "A line drawn tangent to a circle is perpendicular to a radius of the circle drawn to the point of tangency."	G.4B **G.5A** G.5B G.5C G.12A	3
		End of Topic Assessment			1
		Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*			5

Topic 3: Using Congruence Theorems

ELPS: 1.A, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 4.A, 4.B, 4.C, 4.K, 5.B, 5.E

Topic Pacing: 9 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**SSS, SAS, AAS, … S.O.S.!** Using Triangle Congruence to Determine Relationships Between Segments	Students construct a right triangle given line segments representing the length of a leg and the length of the hypotenuse. They conclude that their various constructions result in a unique triangle. Students provide the reasons for statements in a two-column proof of the Hypotenuse-Leg Congruence Theorem and explain the algebraic reasoning used to justify this theorem. Students explain how the Leg-Leg Congruence Theorem and the Leg-Angle Congruence Theorem are equivalent to the SAS, ASA, or AAS Congruence Theorems. The term *tangent segment* is defined, and students use measuring tools to compare the lengths of tangent segments drawn from the same point on the exterior of a circle. The Tangent Segment Theorem is stated, and students analyze a Worked Example containing a proof plan for the theorem.	• The *Hypotenuse-Leg Congruence Theorem* states: "If the hypotenuse and leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent." • The *Leg-Leg Congruence Theorem* states: "If the two corresponding shorter legs of two right triangles are congruent, then the triangles are congruent." • The *Leg-Angle Congruence Theorem* states: "If the leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another triangle, then the triangles are congruent." • The Leg-Leg and Leg-Angle Congruence Theorems can be justified using SSS, SAS, ASA, and/or AAS triangle congruence. • The *Tangent Segment Theorem* states: "If two tangent segments are drawn from the same point on the exterior of a circle, then the tangent segments are congruent."	**G.5A** **G.6B** G.12A	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
2	**Props to You** Properties of Quadrilaterals	Students prove the properties of a parallelogram, a rhombus, a rectangle, a square, a trapezoid, and a kite using formal two-column or paragraph proof formats, as well as informal reasoning. Students apply the theorems to solve problem situations.	• A parallelogram is defined as a quadrilateral with opposite sides parallel. The properties of a parallelogram include: ◦ The opposite sides of a parallelogram are congruent. ◦ The opposite angles of a parallelogram are congruent. ◦ The diagonals of a parallelogram bisect each other. ◦ If one pair of opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram. • A rhombus is defined as a parallelogram with all sides congruent. The properties of a rhombus include: ◦ The diagonals of a rhombus bisect the vertex angles. ◦ The diagonals of a rhombus are perpendicular to each other. • A rectangle is defined as a parallelogram with all angles congruent. The diagonals of a rectangle are congruent. • A square is defined as a parallelogram with all angles congruent and all sides congruent. The square has all of the properties of the rectangle and rhombus. • An isosceles trapezoid is defined as a trapezoid with congruent non-parallel sides. The base angles of an isosceles trapezoid are congruent. • The *Trapezoid Midsegment Theorem* states: "The midsegment of a trapezoid is parallel to each of the bases and its length is one half the sum of the lengths of the bases." • A kite is defined as a quadrilateral with two pairs of consecutive congruent sides. • The properties of a kite include: ◦ One diagonal of a kite is a line of symmetry. ◦ One diagonal of a kite bisects a pair of opposite angles. ◦ One diagonal of a kite is the perpendicular bisector of the other diagonal.	G.6E	3

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Three-Chord Song** Relationships Between Chords	Students explore a problem situation that asks them to determine the diameter of a circular plate given only a broken piece of the plate. Students conjecture about methods that may be used to determine the diameter. They then prove the Diameter–Chord Theorem, Equidistant Chord Theorem, and Equidistant Chord Converse Theorem. They also prove the Congruent Chord–Congruent Arc Theorem and its converse. Finally, students revisit and solve the broken-plate problem from the Getting Started.	• The *Diameter–Chord Theorem* states: "If a circle's diameter is perpendicular to a chord, then the diameter bisects the chord and bisects the arc determined by the chord." • The *Equidistant Chord Theorem* states: "If two chords of the same circle or congruent circles are congruent, then they are equidistant from the center of the circle." • The *Equidistant Chord Converse Theorem* states: "If two chords of the same circle or congruent circles are equidistant from the center of the circle, then the chords are congruent." • The *Congruent Chord–Congruent Arc Theorem* states: "If two chords of the same circle or congruent circles are congruent, then their corresponding arcs are congruent." • The *Congruent Chord–Congruent Arc Converse Theorem* states: "If two arcs of the same circle or congruent circles are congruent, then their corresponding chords are congruent."	G.5B G.5C G.12A	1
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					2

3 Investigating Proportionality

Module Pacing: 26 Days

Topic 1: Similarity

ELPS: 1.A, 1.C, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.K, 5.E

Topic Pacing: 15 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Big Little, Big Little** Dilating Figures to Create Similar Figures	Students perform dilations on triangles and other figures both on and off the coordinate plane. They explore the ratios formed as a result of dilations and recall scale factor. *Similar triangles* are defined, and students explore the relationships between corresponding sides and between corresponding angles. They then use similarity statements to draw similar triangles and use the definition of similarity in terms of transformations to determine whether pairs of figures are similar.	• A *dilation* is a transformation that enlarges, reduces, or keeps congruent a pre-image to create an image. • The center of dilation is a fixed point at which a figure is dilated. • The scale factor of a dilation is the ratio of the distance from the center of dilation to a point on the image to the distance from the center of dilation to the corresponding point on the pre-image. • When the scale factor is greater than 1, the dilation is an enlargement. When the scale factor is between 0 and 1, the dilation is a reduction. When the scale factor is exactly 1, the dilation produces a congruent figure.	G.3A **G.4C** G.7A	**1**
2	**Similar Triangles or Not?** Establishing Triangle Similarity Criteria	Students use proof by construction to prove several theorems related to similar triangles. These theorems include the Angle-Angle Similarity Theorem, the Side-Side-Side Similarity Theorem, and the Side-Angle-Side Similarity Theorem. A graphic organizer is used to summarize the theorems in this lesson.	• The *Angle-Angle Similarity Theorem* states: "If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar." • The *Side-Side-Side Similarity Theorem* states: "If all three corresponding sides of two triangles are proportional, then the triangles are similar." • The *Side-Angle-Side Similarity Theorem* states: "If two of the corresponding sides of two triangles are proportional and the included angles are congruent, then the triangles are similar."	G.5C G.7A **G.7B**	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Keep It in Proportion** Theorems About Proportionality	Students use both paragraph and two-column proofs to prove the Angle Bisector/Proportional Side Theorem, the Triangle Proportionality Theorem, the Converse of the Triangle Proportionality Theorem, the Proportional Segments Theorem, the Triangle Midsegment Theorem, and that the medians of a triangle are concurrent. Each of these theorems is applied in problem situations.	• The *Angle Bisector/Proportional Side Theorem* states: "A bisector of an angle in a triangle divides the opposite side into two segments whose lengths are in the same ratio as the lengths of the sides adjacent to the angle." • The *Triangle Proportionality Theorem* states: "If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally." • The *Converse of the Triangle Proportionality Theorem* states: "If a line divides the two sides of a triangle proportionally, then it is parallel to the third side." • The *Proportional Segments Theorem* states: "If three parallel lines intersect two transversals, then they divide the transversals proportionally." • The *Triangle Midsegment Theorem* states: "The midsegment of a triangle is parallel to the third side of the triangle and half the measure of the third side of the triangle." • The medians of a triangle are concurrent.	**G.2B** G.5B G.5C G.6D G.8A	4
Mid-Topic Assessment					1
4	**This Isn't Your Average Mean** More Similar Triangles	The term *geometric mean* is defined and used in triangle theorems to solve for unknown measurements. Students practice using the Right Triangle/Altitude Similarity Theorem, the Right Triangle Altitude/Hypotenuse Theorem, and the Right Triangle Altitude/Leg Theorem to solve problems. They are then guided through the steps necessary to prove the Pythagorean Theorem using similar triangles.	• The *Right Triangle/Altitude Similarity Theorem* states: "If an altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other." • The *geometric mean* of two numbers a and b is the number x such that $\frac{a}{x} = \frac{x}{b}$. • The *Right Triangle Altitude/Hypotenuse Theorem* states: "The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse." • The R*ight Triangle Altitude/Leg Theorem* states: "If the altitude is drawn to the hypotenuse of a right triangle, each leg of the right triangle is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to the leg." • The Pythagorean Theorem states: "If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$."	G.6D **G.7B** G.8B	1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**Run It Up the Flagpole** Application of Similar Triangles	Advance preparation and materials are needed for this lesson. Indirect measurement is an activity that takes students out of their classroom and school building. Students use similar triangles to measure the height of objects such as flagpoles, tops of trees, telephone poles, or buildings. Each pair of students needs a tape measure, a marker, and a flat pocket mirror. In addition to the outside activity, students are given several situations in which they create proportions related to similar triangles to solve for unknown measurements.	• *Indirect measurement* is the process of using proportions related to similar triangles to determine a measurement when direct measurement is inconvenient or difficult. • The mirror method and the shadow method are used to diagram similar triangles. • The *Law of Reflection* states that the *angle of incidence* is congruent to the *angle of reflection*.	**G.7B**	**1**
6	**Jack's Spare Key** Partitioning Segments in Given Ratios	Students use the Midpoint Formula and the Distance Formula to determine the midpoints of line segments on the coordinate plane. The *midpoint* is described as a point that divides a line segment in a 1:1 ratio. Then, the students expand their thinking to include dividing a line segment in other ratios. Students are guided through different processes of partitioning directed line segments into given ratios and use the Triangle Proportionality Theorem to justify their reasoning. They then partition several line segments into given ratios on the coordinate plane.	• The midpoint of a line segment is the point on the segment that is equidistant from the endpoints of the line segment. • The Midpoint Formula states that the midpoint between any two points on a coordinate plane, (x_1, y_1) and (x_2, y_2), is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. • The Triangle Proportionality Theorem states: "If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally."	G.2A G.5B G.8A	**1**
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					**3**

Topic 2: Trigonometry

ELPS: 1.A, 1.C, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.G, 4.K, 5.E

Topic Pacing: 11 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Three Angle Measure** Introduction to Trigonometry	Students informally explore side length ratios in right triangles. Students construct vertical lines from different points on the hypotenuses of 45°–45°–90° and 30°–60°–90° triangles to the base to form similar right triangles and determine the lengths of the sides. They then convert the lengths into ratios and compare them. Students calculate the slope of the hypotenuse and realize it is the same as the opposite-to-adjacent ratio for both the 45°–45°–90° triangle and the 30°–60°–90° triangle. Students conclude that the ratios they studied are constant in similar right triangles, given the same reference angle. They first use measuring tools to estimate the values of these ratios and then use the Pythagorean Theorem to determine exact values of these ratios. Students describe how the values of these ratios change when the measure of the reference angle increases or decreases.	• Similar right triangles are formed by dropping vertical line segments from the hypotenuse perpendicular to the base of the right triangles. • Given the same reference angle, the ratios $\frac{\text{side opposite to reference angle}}{\text{hypotenuse}}$, $\frac{\text{side adjacent to reference angle}}{\text{hypotenuse}}$, and $\frac{\text{side opposite to reference angle}}{\text{side adjacent to reference angle}}$ are constant. • The side length ratios $\frac{\text{side opposite to reference angle}}{\text{hypotenuse}}$, $\frac{\text{side adjacent to reference angle}}{\text{hypotenuse}}$, and $\frac{\text{side opposite to reference angle}}{\text{side adjacent to reference angle}}$ are the same for all 45°–45°–90° triangles given the same reference angle. • The side length ratios $\frac{\text{side opposite to reference angle}}{\text{hypotenuse}}$, $\frac{\text{side adjacent to reference angle}}{\text{hypotenuse}}$, and $\frac{\text{side opposite to reference angle}}{\text{side adjacent to reference angle}}$ are the same for all 30°–60°–90° triangles given the same reference angle. • The slope of the hypotenuse of a 45°–45°–90° triangle and the $\frac{\text{side opposite to reference angle}}{\text{side adjacent to reference angle}}$ ratio are equal to 1. • The slope of the hypotenuse of a 45°–45°–90° triangle and a 30°–60°–90° triangle is equal to the $\frac{\text{side opposite to reference angle}}{\text{side adjacent to reference angle}}$ ratio. • The Pythagorean Theorem can be used to determine the exact ratios of side lengths in similar right triangles.	**G.9A** **G.9B**	1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
2	**Going on a Tangent** Tangent Ratio and Inverse Tangent	The terms *tangent* and *inverse tangent* are defined, and tangent is explicitly connected to the concept of slope. A real-world context is provided, which requires students to determine the length of the hypotenuse and slope in right triangles. Applying the tangent ratio to similar triangles, students conclude that the value of the tangent of congruent angles of similar triangles is always the same and as the value of the tangent increases, the measure of an acute angle increases. They write expressions based on the complementary relationship between the two acute angles in right triangles. When the inverse tangent is introduced, students use calculators to solve for the measure of an acute angle in a right triangle. Calculators are used throughout this lesson to compute the value of the ratios.	• The *tangent (tan)* of an acute angle in a right triangle is the ratio of the side length that is opposite the angle to the side length that is adjacent to the angle. • The *inverse tangent (arctangent)* of x is the measure of an acute angle whose tangent is x.	**G.9A**	2
3	**Show Me a Sine** Sine Ratio and Inverse Sine	The terms *sine* and *inverse sine* are defined. A real-world context is given for determining the sine ratio in right triangles. Students conclude that as the reference angle increases in measure, the sine ratio increases in value. They also conclude that the value of sine is always less than 1 because the hypotenuse (the denominator in the sine ratio) is the longest side of the right triangle. When the inverse sine is introduced, students use their calculators to solve for the measure of an acute angle in a right triangle. Calculators are used throughout this lesson to compute the value of the ratios.	• In a right triangle, the ratio $\frac{\text{side opposite to reference angle}}{\text{hypotenuse}}$ increases as the reference angle increases. • The *sine (sin)* of an acute angle in a right triangle is the ratio of the length of the side that is opposite the angle to the length of the hypotenuse. • The *inverse sine (or arcsine)* of x is the measure of an acute angle whose sine is x.	**G.9A**	**1**
Mid-Topic Assessment					1
4	**Can I Get a Cosine?** Cosine Ratio and Inverse Cosine	The terms *cosine* and *inverse cosine* are introduced. A real-world context is given for determining the cosine ratio in right triangles. Students conclude that as the reference angle increases in measure, the cosine ratio decreases in value and the value of cosine is always less than 1 because the hypotenuse (the denominator in the cosine ratio) is the longest side of the right triangle. When the inverse cosine is introduced, students use their calculators to solve for the measure of an acute angle in a right triangle. Calculators are used throughout this lesson to compute the value of the ratios.	• The *cosine (cos)* of an acute angle in a right triangle is the ratio of the length of the side that is adjacent to the angle to the length of the hypotenuse. • The *inverse cosine (or arccosine)* of x is the measure of an acute angle whose cosine is x.	**G.9A**	**1**

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**Fishing for Complements** Complement Angle Relationships	Students explore the complementary relationships involved with trigonometric ratios and use them to solve real-world problem situations. They use complementary angle relationships, their knowledge of the side relationships of special right triangles, and the Pythagorean Theorem to determine the values of the sine and cosine ratios of a 45° angle, a 30° angle, and a 60° angle.	• When $\angle A$ and $\angle B$ are acute angles in a right triangle, $\sin \angle A = \cos \angle B$ and $\cos \angle A = \sin \angle B$.	**G.9A** **G.9B**	1
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					3

4 Connecting Geometric and Algebraic Descriptions

Module Pacing: 16 Days

Topic 1: Circles and Volume

ELPS: 1.A, 1.C, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.G, 4.K, 5.E

Topic Pacing: 12 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**All Circles Great and Small** Similarity Relationships in Circles	Students construct arguments, one using similarity transformations and one using proportional relationships, to show that all circles are similar. Arc length is distinguished from arc measure, and students use given formulas to solve real-world problem situations involving arc lengths. The term *radian* is defined. Students explore the proportionality between the length of the intercepted arc of the central angle and the radius of the circle and determine that the radian measure of the angle is the constant of proportionality. They solve for arc length using the radian measure of a central angle. Students convert degree measures to radians and radians to degree measures in different situations.	• The radius of a circle, r, maps onto the circumference of the circle 2π times. • All circles are similar figures. • There is a proportional relationship between the measure of an arc length of a circle, s, and the circumference of the circle. • The formula for *arc length* can be written as $s = \frac{m}{360°}(2\pi r)$, where s is the arc length and m is the central angle measure. • A *radian* is the measure of a central angle whose arc length is the same as the radius of the circle. • The formula for arc length can be written as $s = \theta r$, where s is the arc length and θ is the central angle measure in radians. • When converting degrees to radians, multiply a degree measure by $\frac{\pi}{180°}$. When converting radians to degrees, multiply a radian measure by $\frac{180°}{\pi}$.	G.12B G.12D	**1**

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
2	**A Piece of Pi** Sectors and Segments of a Circle	Students explore and describe methods for determining the area of a sector and the area of a segment of a circle. The formula for each is stated and students apply them to solve problem situations.	• A *sector of a circle* is a region of the circle bounded by two radii and the included arc. • The area for the sector of a circle can be determined by multiplying the area of the circle, $A = \pi r^2$, by the fraction $\frac{m}{360°}$, where m represents the central angle measure of the sector. • The area for a sector of a circle can be determined by the formula $A_{sector} = \frac{m}{360°}(\pi r^2)$. • The *segment of a circle* is a region of the circle bounded by a chord and the included arc. Each segment of a circle can be associated with a sector of the circle. • The strategy for calculating the area of a segment of a circle is to calculate the area of the sector associated with the segment and from that, subtract the area of the triangle within the sector formed by the two radii and the chord or $A_{segment} = A_{sector} - A_{triangle}$. • The area for a segment of a circle can be determined by the formula $A_{segment} = \frac{m}{360°}(\pi r^2) - \frac{1}{2}(b)(h)$.	**G.11B** G.12C	1
		Mid-Topic Assessment			1
3	**Do Me a Solid** Building Three-Dimensional Figures	Students use transformations of two-dimensional figures to create three-dimensional figures. They create a cylinder, sphere, and cone by rotating a rectangle, circle, and triangle, respectively, about a line segment. Students also use vertical translations of a circle and polygons to create a cylinder and right prisms, respectively. They use isometric paper to visualize three-dimensional figures on a two-dimensional plane. Students then explore an oblique prism and an oblique cylinder as translations of a polygon or circle in a direction not perpendicular to the plane in which they lie. They use stacking (or vertical translations) to develop the volume formula for a cylinder and a general formula that applies to all prisms, $V = Bh$, where B is the area of the base and h is the height. They use Cavalieri's Principle to make sense of the fact that the same general volume formula, $V = Bh$, also applies to oblique cylinders and oblique prisms.	• Rigid motion is used in the process of redrawing two-dimensional plane figures as three-dimensional solids. • Models of three-dimensional solids are formed using rotations and translations of plane figures through space. • Models of two-dimensional plane figures can be rotated or stacked to create models of three-dimensional solids. • The volume formula, $V = Bh$, where B is the area of the base and h is the height, applies to all right and oblique cylinders and right and oblique prisms. • *Cavalieri's Principle* for area states that if the lengths of one-dimensional slices—just line segments—of two figures are the same, then the figures have the same area. • Cavalieri's Principle for volume states that, given two solids included between parallel planes, if every plane cross section parallel to the given planes has the same area in both solids, then the volumes of the solids are equal.	G.10A **G.11D**	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
4	**Get to the Point** Building Volume and Surface Area Formulas for Pyramids, Cones, and Spheres	Students stack similar two-dimensional figures to create three-dimensional figures. They observe the changing volume of a pyramid and a prism with congruent bases and the same height and conclude that a pyramid has a volume that is one third the volume of a prism with the same base and height. Students analyze a Worked Example that uses similar triangles to show that the areas of coplanar cross-sections of a right circular cone and a square pyramid are equal. They conclude that the formula for the volume of a cone is $V = \frac{1}{3}\pi r^2$. Students build two different models of a cylinder using two pieces of 8.5 in. by 11 in. paper to compare the volumes of the figures created. Students determine the lateral surface areas and total surface areas of various three-dimensional solid figures and investigate the surface area formulas related to each figure. The formulas for the volume of a sphere and the surface area of a sphere are given, and students use the formulas to answer questions related to the dimensions of different spheres. They solve real-world problems using reasoning and the formulas presented in the lesson.	• The volume formula for a pyramid is $V = \frac{1}{3}Bh$, where V is the volume, B is the area of the base, and h is the height of the pyramid. • The volume of a pyramid is one-third the volume of a prism with the same base area and height. • The volume formula for a cone is $V = \frac{1}{3}\pi r^2 h$, where V is the volume, r is the length of the radius of the base, and h is the height of the cone. • The volume of a cone is one-third the volume of a cylinder with the same base area and height. • The *lateral surface area* of a three-dimensional figure is the sum of the areas of its lateral faces. • The *total surface area* of a three-dimensional figure is the sum of its bases and lateral faces. • The volume formula for a sphere is $V = \frac{4}{3}\pi r^3$, where V is the volume and r is the length of the radius of the sphere. • The total surface area of a sphere is $4\pi r^2$.	**G.10B** **G.11C** **G.11D**	3
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					3

Topic 2: Circles and Cross-Sections

ELPS: 1.A, 1.C, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.G, 4.K, 5.E

Topic Pacing: 4 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Give Me a Slice** Cross-Sections	Students reason about when the intersection of a plane and a geometric solid creates a cross-section that is a single point or a line segment. A variety of cross-sections are created as geometric solids are sliced with planes drawn parallel to the base, perpendicular to the base, and on an angle to the base. Students practice identifying solids given their cross-sections, and cross-sections given their solids.	• A cross-section of a three-dimensional solid can be a point, a line segment, or a two-dimensional figure that is formed by the intersection of the solid and a plane. • The maximum number of sides of a cross-section equals the number of sides of faces of the solid, if it is a polyhedron.	G.10A	1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
2	**X^2 Plus Y^2 Equals Radius^2** Deriving the Equation for a Circle	Students derive the standard form for the equation of a circle, with the center point (*h*, *k*) and the radius *r*, using the Pythagorean Theorem. Given the radius and the center point, they practice writing the equation for a circle in standard form. The general form for the equation of a circle is then provided. A Worked Example shows how to rewrite an equation in general form as an equation in standard form by completing the square. The advantage of having the equation of a circle written in standard form is the ease with which the radius and center point can be identified.	• The standard form of the equation of a circle centered at (h, k) with radius r can be expressed as $(x - h)^2 + (y - k)^2 = r^2$. • The equation for a circle in general form is $Ax^2 + Cy^2 + Dx + Ey + F = 0$, where $A, C, D, E,$ and F are constants, $A = C$, and $x \neq y$. • An equation written in general form can be rewritten in standard form using the algebraic procedure called *completing the square*.	G.12E	**1**
3	**A Blip on the Radar** Determining Points on a Circle	Students use the Pythagorean Theorem and the Distance Formula to determine whether a point lies on a circle. They consider circles where the location of the center point is either at the origin or not at the origin and the coordinates of a point at various locations on the circle. They use what they know about equations of circles to solve real-world problems.	• The Pythagorean Theorem can be used to determine whether a point lies on the circumference of a circle when the center point is located at the origin and the length of the radius is given. • The Pythagorean Theorem, the Distance Formula, and symmetry can be used to determine whether a point lies on the circumference of a circle when the center point is not located at the origin and the coordinates of a point on the circumference of a circle are given. • The coordinates of the points at which a circle and line intersect can be determined algebraically by writing equations for the line and for the circle, substituting the expression representing the *y*-value of the line into the equation of the circle, and then solving the quadratic equation. • Segments drawn tangent to the same circle from the same exterior point are congruent. • The equation of a line drawn tangent to a circle can be determined given the center point of the circle, a radius drawn to the point of tangency, and the coordinates of the point of tangency.	**G.2B** G.12E	**0**
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					**1**

5 Making Informed Decisions

Module Pacing: 15 Days

Topic 1: Independence and Conditional Probability

ELPS: 1.A, 1.B, 1.C, 1.D, 1.E, 1.G, 2.A, 2.B, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 3.H, 3.I, 4.A, 4.B, 4.C, 4.D, 4.E, 4.F, 4.G, 4.I, 4.J, 4.K, 5.A, 5.C, 5.E, 5.F, 5.G

Topic Pacing: 6 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**What Are the Chances?** Compound Sample Spaces	Students learn strategies for determining the sample space of compound events. They begin with a review of terms associated with probability such as *sample space, event,* and *probability model*. For a given situation, students list the sample space, construct a probability model, and differentiate between uniform and non-uniform probability models. Tree diagrams are then introduced as a way to list outcomes in a situation. Students examine examples of tree diagrams, and then, for different situations, create their own tree diagrams and organized lists of the corresponding sample space. They analyze the sample space in each situation, distinguishing between situations that involve independent events from disjoint sets and dependent events from intersecting sets. Students identify a situation associated with dependent events that does not allow repetition. The Counting Principle is stated and discussed as an efficient method for determining the number of outcomes in a sample space without listing each possible outcome.	• The *probability* of an event is the ratio of the number of desired outcomes to the total number of possible outcomes. • An *outcome* is a result of an experiment. A sample space is all of the possible outcomes in a probability situation. An *event* is an outcome or set of outcomes in a sample space. • A *probability model* lists the possible outcomes and the probability of each outcome. The sum of the probabilities in the model equals one. • The *complement of an event* is an event that contains all the outcomes in the sample space that are not outcomes in the event. • A *non-uniform probability model* is a model in which all of the outcomes are not equal. • *Disjoint sets* do not have common elements. Intersecting sets have at least one common *element*. • *Independent events* are events for which the occurrence of one event has no impact on the occurrence of the other event. *Dependent events* are events for which the occurrence of one event has an impact on the occurrence of the following events. • The *Counting Principle* states: "If an action *A* can occur in *m* ways and for each of these *m* ways, an action *B* can occur in *n* ways, then actions *A* and *B* can occur in $m \cdot n$ ways."	**G.13C**	2

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
2	**And?** Compound Probability with *And*	Students determine the probability of two or more independent events and two or more dependent events. The Rule of Compound Probability involving *and* is stated and used to compute compound probabilities. Several situations present students with the opportunities to construct tree diagrams, create organized lists, and compute the probabilities of either independent or dependent compound events. For dependent compound events, the term *conditional probability* is defined, and students use a different notation for the Rule of Compound Probability involving *and* to calculate probabilities.	• A *compound event* is an event that consists of two or more events. • The *Rule of Compound Probability* involving *and* states: "If Event A and Event B are independent, then the probability that Event A happens and Event B happens is the product of the probability that Event A happens and the probability that Event B happens, given that Event A has happened." However, A happening has no influence on B happening. Using probability notation, the Rule of Compound Probability involving *and* is $P(A \text{ and } B) = P(A) \cdot P(B)$. • The Rule of Compound Probability involving *and* states: "If Event A and Event B are dependent, then the probability that Event A happens and Event B happens is the product of the probability that Event A happens and the probability that Event B happens, given that Event A has happened." Using probability notation, the Rule of Compound Probability involving *and* is $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.	**G.13C** G.13D	1
3	**Or?** Compound Probability with *Or*	Students determine the probability of one or another independent event and the probability of one or another dependent event. The Addition Rule for Probability is stated and used to compute probabilities. Several situations present students with the opportunities to construct tree diagrams, create organized lists, complete tables, and compute $P(A)$, $P(B)$, $P(A \text{ and } B)$, and $P(A \text{ or } B)$ with respect to the problem situation. Students create a graphic organizer to record the different types of compound events they have studied: independent events $P(A \text{ and } B)$, independent events $P(A \text{ or } B)$, dependent events $P(A \text{ and } B)$, and dependent events $P(A \text{ or } B)$.	• A compound event is an event that consists of two or more events. • The *Addition Rule for Probability* states: "The probability that Event A occurs or Event B occurs is the probability that Event A occurs plus the probability that Event B occurs minus the probability that both A and B occur." Using probability notation, the Addition Rule for Probability is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.	**G.13C**	1
4	**And, Or, and More!** Calculating Compound Probability	Students analyze scenarios involving a standard deck of playing cards, choosing committee members, and a menu and determine compound probabilities. Students determine the probability of independent events $P(A \text{ and } B)$ with replacement, independent events $P(A \text{ or } B)$ with replacement, dependent events $P(A \text{ and } B)$ without replacement, and dependent events $P(A \text{ or } B)$ without replacement.	• Situations "with replacement" generally involve independent events. Whether or not the first event happens has no effect on the second event. • Situations "without replacement" generally involve dependent events. If the first event occurs, it has an impact on the probability of subsequent events.	**G.13C**	**0**
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					**1**

Topic 2: Computing Probabilities

ELPS: 1.A, 1.D, 1.E, 1.F, 1.G, 2.C, 2.D, 2.F, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 3.G, 3.J, 4.A, 4.B, 4.C, 4.G, 4.H, 4.K, 5.D, 5.E

Topic Pacing: 9 Days

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
1	**Table Talk** Compound Probability for Data Displayed in Two-Way Tables	Rolling two number cubes and the results of surveys are contexts used to create sample spaces, organized lists, and tables. The converse of the multiplication rule is stated and used to determine when events are independent. The terms *frequency, frequency table, two-way frequency table, relative frequency,* and *two-way relative frequency table* are introduced. Students complete these types of tables and use the tables to answer questions related to the situations.	• A *two-way table* is a table that shows the relationship between two data sets, one organized in rows and one organized in columns. • A *frequency table* is a table that shows the frequency of an item, number, or event appearing in a sample space. • A *two-way frequency table,* or *contingency table,* shows the number of data points and their frequencies for two variables. • A *relative frequency* is the ratio of occurrences within a category to the total number of occurrences. • A *two-way relative frequency table* displays the relative frequencies for two categories of data. • Two-way tables can be used to determine the probabilities of compound events. • The converse of the multiplication rule for probability states: "If the probability of two events A and B occurring together is $P(A) \cdot P(B)$, then the two events are independent."	**G.13C** G.13E	1
2	**It All Depends** Conditional Probability	Rolling two number cubes and calculating the sum is once again used to generate a two-way data table listing the possible outcomes. Different events are described and students calculate $P(A)$, $P(B)$, and $P(A \text{ and } B)$. Students derive a formula for computer conditional probability, $(P(B\|A) = \frac{P(A \text{ and } B)}{P(A)}$. The conditional probability formula is applied to several different situations.	• *Conditional probability* is the probability of Event B, given that Event A has already occurred. • The notation for conditional probability is $P(B\|A)$, which reads, "The probability of Event B, given Event A." • When $P(B\|A) = P(B)$, the two events, A and B, are independent. • When $P(B\|A) \neq P(B)$, the two events, A and B, are dependent. • The conditional probability formula is stated as $\frac{P(B \text{ and } A)}{P(A)}$.	**G.13C** G.13D G.13E	1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
3	**Give Me 5!** Permutations and Combinations	The terms *factorial, permutation*, and *combination* are defined. Students derive the formulas to calculate permutations and combinations, then apply them in different situations. Situations involve permutations with and without repeated elements. Students answer questions, complete tables, and make the connections necessary to develop additional formulas related to combinations and permutations. Circular permutations are introduced. Students conclude that the formula for the circular permutation of n objects is $(n - 1)!$.	• The *factorial* of n, which is written with an exclamation mark as $n!$, is the product of all non-negative integers less than or equal to n: $n(n - 1)(n - 2)\ldots$ • A *permutation* is an ordered arrangement of items without repetition. • The notation denoting a permutation of r elements taken from a collection of n items is ${}_nP_r = P(n, r) = P^n_r$. • The formula used to compute the number of permutations, P, of r elements chosen from n elements is ${}_nP_r = \frac{n!}{(n - r)!}$. • A *combination* is an unordered collection of items. • The notation denoting a combination of r elements taken from a collection of n elements is ${}_nC_r = C(n, r) = C^n_r$. • The formula used to compute the number of combinations, C, of r elements chosen from n elements is ${}_nC_r = \frac{n!}{(n - r)!r!}$. • The formula for the number of permutations of n elements with k copies of an element is $\frac{n!}{k!}$. • The *circular permutation* of n objects is $(n - 1)!$.	G.13A	2
4	**A Different Kind of Court Trial** Independent Trials	Situations in this lesson focus on multiple trials for two independent events. Making free throw shots, rolling number cubes, and rolling a tetrahedron are used to generate the probabilities of two independent events. Outcomes are organized in a table and the table is connected to Pascal's Triangle. Students use Pascal's Triangle to compute the probability of an occurrence. A formula using combinations is applied to different situations to calculate probabilities for two independent events over multiple trials.	If the probability of Event A is p and the probability of Event B is $1 - p$, then the probability of Event A occurring r times and Event B occurring $n - r$ times in n trials is P(A occurring r times and B occurring $n - r$ times) or ${}_nC_r(P)^r(1 - P)^{n-r}$.	G.13A	1

Lesson	Lesson Title	Highlights	Essential Ideas	TEKS*	Pacing**
5	**What Do You Expect?** Expected Value	The terms *geometric probability* and *expected value* are introduced in this lesson. Geometric probability is applied to dartboards containing geometric shapes. Expected value is applied to money wheels divided into eight equal regions.	• *Geometric probability* is the likelihood of an event occurring based on geometric relationships such as length, area, or volume. • *Expected value* is the sum of the values of a random variable with each value multiplied by its probability of occurrence.	G.13B	**1**
End of Topic Assessment					1
Learning Individually with Skills Practice or MATHia *Schedule these days strategically throughout the topic to support student learning.*					**2**

Total Days: 150

Learning Together: 97

Learning Individually: 37

Assessments: 16