1

Put Your Input In, Take Your Output Out

Geometric Components of Rigid Motions

MATERIALS

Compasses
Patty paper
Protractors (optional)
Straightedges

Lesson Overview

Students develop the concept that geometric rigid motion transformations can be considered as functions, with rotations, reflections, and translations as the operations. Translations can be described using lines and line segments. Reflections can be described using lines. Rotations can be described using rotation angles. The inputs and outputs are geometric shapes. Each input and its corresponding output have the same size and shape.

Geometry

Coordinate and Transformational Geometry

(3) The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity).
The student is expected to:

(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.

(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- Pre-images transformed by rigid motions such as translations, reflections, and rotations are congruent to their images.
- Translations of lines produce parallel lines.
- Points and lines are essential building blocks of geometry and of geometric transformations.
- A line is a geometric object such that if any part of the line is translated to another part of the line so that the two parts have two points in common, then the first part will lie exactly on top of the second part.
- Translations can be described using lines and line segments. Reflections can be described using lines. Rotations can be described using rotation angles.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Transformation Machine

Students are introduced to transformations as functions by representing each of three rigid motions (translation, reflection, and rotation) as a machine that takes a shape as an input and produces a congruent shape as an output. They then draw another input shape and output shape to represent each rigid motion.

Develop

Activity 1.1: Lines, Line Segments, and Angles

Students either review or are introduced to terms necessary to experiment with transformations in the plane. They also address both positive and negative angles of rotation.

Demonstrate

Talk the Talk: Shake It All About

Students are given a transformation machine comprised of line segments, figures with and without center points, and two target shapes. To use the transformation machine students must first trace one of the two target shapes onto patty paper. They then determine a path for each of the input shapes, a triangle and a square, to move from the start line through the machine and map back onto itself. Students then describe the sequences of transformations they used.

Facilitation Notes

In this activity, students are introduced to transformations as functions by representing each of three rigid motions (translation, reflection, and rotation) as a machine that takes a shape as an input and produces a congruent shape as an output. They then draw another input shape and output shape to represent each rigid motion.

Ask a student to read the introduction. Discuss as a class.

Questions to ask

- What is an example of a function machine?
- What is the input of a function machine? What is its output?
- What is meant by a transformation machine?
- What is the input of a transformation machine? What is its output?
- If it is a rigid motion transformation machine, what operations can it perform?

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

To assist all students,

- Discuss the machine in Question 1, part (a) as a class.
- Have students use patty paper to demonstrate the operations in the left column before creating their own input shape.

As students work, look for

- Use of the terms *rotation, reflection,* and *translation*.
- Detailed responses that include direction and degree measure of the rotation, the orientation of the line of reflection, and the horizontal and vertical nature of the translation.
- Different shapes used to model the same transformation machine.

Misconception

Confusion as to why the output in Question 1, part (c) remains inside the box. The translation is a specified distance. When the bottom right vertex of the triangle is translated the specific distance of the line segment, it remains inside the box, and the other points on the triangle lie behind it, translated the same distance.

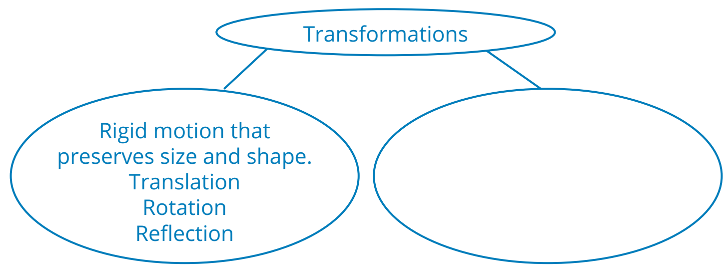
Questions to ask

- What shape did you use?
- What helped you determine that the transformation was a rotation?
- Is the rotation clockwise or counterclockwise? How do you know?
- How many degrees is the figure rotated? How do you know?

- What does the arrow in this transformation machine suggest?
- What does the vertical line in the middle of this machine suggest?
- What does the diagonal line in this transformation machine suggest?
- Do all of these transformation machines preserve size?
- Do all of these transformation machines preserve shape?

Differentiation strategy

To extend the activity, provide a framework to summarize the vocabulary used in this lesson and contrast rigid motions from dilations.



Summary

Geometric rigid motion transformations can be considered as functions, with rotations, reflections, and translations as the operations. The inputs and outputs are geometric shapes. Each input and its corresponding output have the same size and shape.

DEVELOP

Activity 1.1

Lines, Line Segments, and Angles

Facilitation Notes

In this activity, students either review or are introduced to terms necessary to experiment with transformations in the plane. They also address both positive and negative angles of rotation.

Ask a student to read the introduction and definitions aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- Does it matter where the arrows are placed on the translated line? Why or why not?
- Why doesn't the output change?
- How would this question be different if a point was labeled on the line?
- Demonstrate the input and output if a point was placed on the line.

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

Questions to ask

- Is a point an actual dot? How is a point named?
- Does a line have length and width? How is a line named?
- How many line segments are contained on a line?
- How are a line and a line segment different?
- What do a line and a line segment have in common?
- Are two points always collinear? Why or why not?
- How can patty paper be used to show that the line segments in Question 1, part (b) in the previous activity are congruent?

Misconception

Students may not understand the difference between describing and defining a geometric term. A discussion about undefined terms and defined terms may be helpful. The three undefined terms in geometry are *point*, *line*, and *plane*. These undefined terms are the building blocks of geometry and they do not have dimension. All other terms are defined with an understanding of these basic elements.

Ask a student to read the definitions following Question 3 aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

Misconception

It may not be intuitive that positive rotation angles turn counterclockwise and negative rotation angles turn clockwise. This concept aligns with angle placement on the coordinate plane. A positive rotation angle with one ray at 0° opens up from the *x*-axis towards the *y*-axis in the first quadrant.

Questions to ask

- What do a ray and a line have in common?
- How is a ray different than a line?
- Why do you think a rotation angle is based on a circle?

Differentiation strategies

To extend the activity,

- Have students use a protractor to verify that the measures of the angles in the diagrams are 45° and 60°.
- Have students investigate what a 45° angle and a 60° angle look like on the coordinate plane.
- What benchmark angle(s) did you use to identify the 270° angle?
- How do you know whether the angle measure is positive or negative?

Summary

Translations can be described using lines and line segments. Reflections can be described using lines. Rotations can be described using rotation angles.

DEMONSTRATE

Talk the Talk: Shake It All About

Facilitation Notes

In this activity, students are given a transformation machine comprised of line segments, figures with and without center points, and two target shapes. To use the transformation machine students must first trace one of the two target shapes onto patty paper. They then determine a path for each of the input shapes, a triangle and a square, to move from the start line through the machine and map back onto itself. Students then describe the sequences of transformations they used.

Ask a student to read the introduction and rules aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

To scaffold support,

- Discuss the components of the transformation machine.
- Complete an example as a class.

Questions to ask regarding the components of the transformation machine

- What does a vertical translation down look like on this machine?
- What does a vertical reflection look like on this machine?
- What does a horizontal reflection look like on this machine?
- What transformation does the dotted line on the trapezoid suggest?
- Does the dotted line on the trapezoid suggest a vertical or horizontal reflection?
- What transformation does the dotted line on the octagon suggest?
- Does the dotted line on the octagon suggest a vertical or horizontal reflection?
- What transformation does the dotted line on the triangle suggest?
- Does the dotted line on the triangle suggest a vertical or horizontal reflection?

- What transformation does the dotted line on the rectangle suggest?
- Does the dotted line on the rectangle suggest a vertical or horizontal reflection?
- How many locations are available on the start line?

Misconception

Students may not understand that a reflection carries the entire image across the line of reflection. The original figure can be traced across the line of reflection, and its sides can be used for translations.

Questions to ask regarding a transformation of the triangle

- Which of the three possible start locations can lead your shape to the triangle?
- If you place your patty paper image of the triangle on the third (last) start position and slide it down the diagonal guideline, what transformation must it undergo when it moves through the rectangle?
- If the rectangle causes a horizontal reflection, what should be the orientation of the triangle before it undergoes this transformation?
- If the pre-image of the triangle is initially oriented on the start line as a horizontal reflection of the image, and then slides down the diagonal guideline, and the rectangle reflects it across the dotted line, will the triangle be in the correct position to be translated up to the position of the image?

Questions to ask regarding a transformation of the square

- Which of the three possible start locations can lead your shape to the square?
- If you place your patty paper image of the square on the third (last) start position and slide it down the diagonal guideline, and then slide it down the vertical guideline, can you position the side of the square to line up with the side of the triangle?
- What transformation must it undergo when it moves through the triangle?
- If the triangle causes a horizontal reflection, what should be the orientation of the square before it undergoes this transformation?
- If the pre-image of the square is initially oriented on the start line the same way the image is oriented, and then slides down the diagonal, then vertical guideline, and the triangle reflects it across the dotted line, will the square be in the correct position to be translated up to the position of the image?

Summary

A combination of rigid motion transformations can map a figure back onto itself.

NOTES

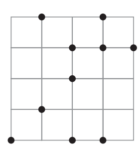
1

Put Your Input In, Take Your Output Out

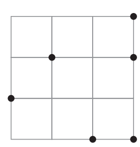
Geometric Components of Rigid Motions

Warm Up

1. Identify the vertices that form a rectangle. Explain why the figure is a rectangle.



2. Identify four vertices that form a trapezoid. Explain why the figure is a trapezoid.



Learning Goals

- Know precise definitions of line segment, angle, and distance along a line.
- Translate lines to produce parallel lines.

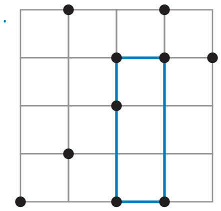
Key Terms

- collinear points
- angle
- ray
- rotation angle

You know a lot about rigid motions, such as translations, reflections, and rotations. How do you use straight lines and angles to represent rigid motion transformations?

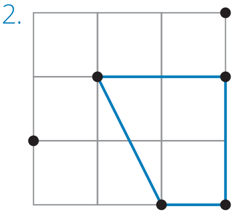
Warm Up Answers

1.



A rectangle is a quadrilateral with two pairs of parallel sides and 4 right angles.

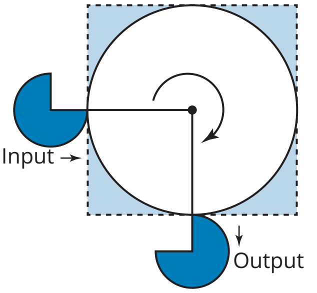
2.



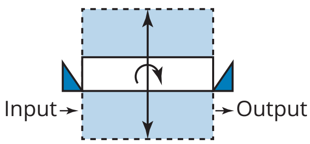
A trapezoid is a quadrilateral with at least one pair of parallel sides.

Answers

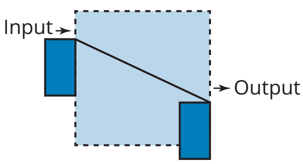
1a. Sample answer.



1b. Sample answer.



1c. Sample answer.



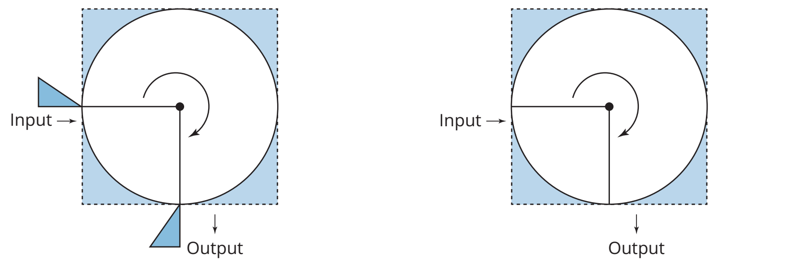
GETTING STARTED

Transformation Machine

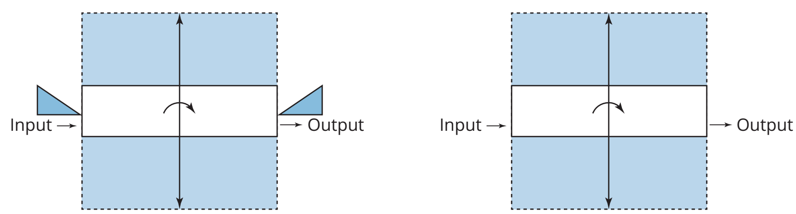
You have learned about function machines, which take a value as input and output another value. In this topic, you will learn to think about geometric rigid motions as functions. These functions can be represented with function machines as well, or transformation machines.

1. Each transformation machine on the left shows a different rigid motion. For each transformation machine on the right, draw an input shape on patty paper. Then apply the transformation and draw the output shape.

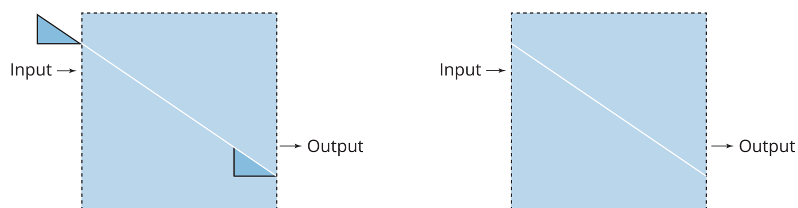
a.



b.



c.



2. Identify the rigid motion represented by each transformation machine.

3. Describe each transformation function. Explain how each input shape is "carried" by geometric objects in the transformation machine to result in the output shape.

Answers

2. Rotation; reflection; translation
3. The input to each function was a shape. In the rotation, the circle carried the input shape and rotated it 270 degrees clockwise to become the output shape. In the reflection, the side of a rectangle carried the input shape and reflected it across a vertical line to become the output shape. In the translation, a line segment carried the input shape to translate it along the line segment and become the output shape.

Answer

1a. The output is a vertical line translated along the line segment. The output line is parallel to the input line.

Remember:

A plane extends infinitely in all directions in two dimensions and has no thickness.

ACTIVITY **1.1**

Lines, Line Segments, and Angles

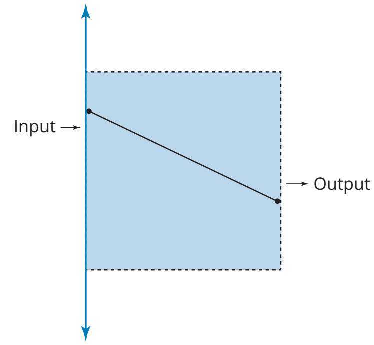
Points and lines are essential building blocks of geometry. They are called undefined terms.

Recall that a point in geometry has no size or shape, but it is often represented using a dot. In a diagram, a point can be labeled using a capital letter. A line is described as a straight, continuous arrangement of an infinite number of points. A line has an infinite length, but no width. Arrowheads are used to indicate that a line extends infinitely in opposite directions. In a diagram, a line can be labeled with a lowercase letter positioned next to the arrowhead.

Points that lie along the same line are called **collinear points**. Recall that a line segment is a part of a line between two points on the line, called the endpoints. A distance along a line is the length of a line segment connecting two points on the line. A line segment *AB* has the distance *AB*.

1. Consider the translation machine from the Getting Started.

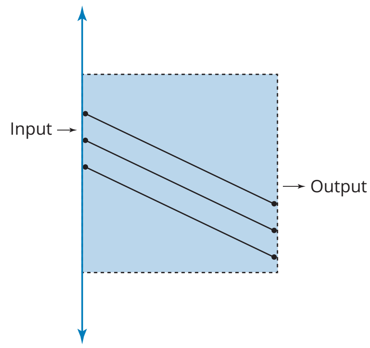
a. Suppose that the input to the machine is a line as shown. Describe the output of the translation machine.



ELL Tip

Determine whether students are familiar with the term *essential*. If not, define *essential* in its adjective form as *absolutely necessary*. Provide examples of synonyms for *essential*, such as *important, crucial, mandatory, critical*, and *vital*. Read aloud the first sentence in the activity, “Points and lines are *essential* building blocks in geometry.” Discuss the meaning of *essential* in the context of this sentence. Clarify any remaining misunderstandings students may have about the term.

b. Suppose that the translation machine is a set of parallel line segments as shown. How does this change the output of the machine when the input is a line?



2. Identify the line segments and distances that were used in the translation machines in the previous activity.

3. Are the line segments in the translation machine in Question 1, part (b), congruent? Use patty paper to justify your answers, and explain your reasoning.

Remember:

Congruent line segments are line segments that have the same length. They represent equal distances.

Answers

1b. It does not change the output.

2. The first translation machine used a line segment inside the machine. The second translation machine used three line segments. Each line segment represents the same distance.

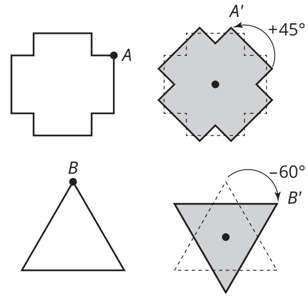
3. Yes. All the line segments are congruent.

Answer

4. −270°

An **angle** is a set of points consisting of a vertex point and two rays extending from the vertex point. A **ray** is a portion of a line that begins with a single point and extends infinitely in one direction.

A **rotation angle** is a directed angle based on a circle. Positive rotation angles turn counterclockwise, and negative rotation angles turn clockwise.



4. Identify the rotation angle that was used in the rotation machine in the Getting Started.

NOTES

TALK the TALK

Shake It All About

The diagram on the next page shows a transformation machine. The transformation machine is designed to provide a path for each of the input shapes, a triangle and a square, to move from the start line through the machine and map back onto itself. The transformation is comprised of line segments, figures with and without center points, and two target shapes. To use the transformation machine you must first trace the target shape onto patty paper.

The transformation machine has these rules:

- The elements in the transformation machine provide ways to move your input shape. Your input shape must be connected to any of the lines or figures in the transformation machine by at least one vertex in order for it to be translated, rotated, or reflected.
- Any dashed or solid line allows you to translate your input shape.
- Any figure with a solid center point allows you to rotate your input shape around that center. The figure carries your input shape around the rotation.
- Any figure with a dashed line allows you to reflect your input shape across that line. The figure carries your input shape across the reflection.

Use the larger diagram of the transformation machine located at the end of the lesson.

1. Copy one of the target shapes onto patty paper. Place the input shape on the start line in an orientation of your choosing. Then determine a sequence of translations, reflections, and rotations that maps the input shape onto the corresponding target shape.

Answer

1. Answers will vary. Check students' work.

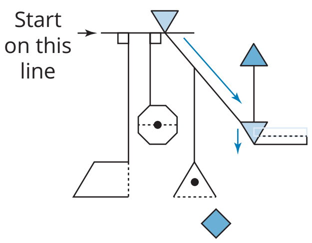
ELL Tip

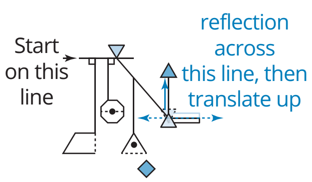
Read aloud the third sentence in the activity, "The transformation is *comprised* of line segments, figures with and without center points, and two target shapes." Discuss the term *comprised* and assess students' prior knowledge to determine their understanding of the term in the context of the sentence. Define *comprise* as *made up of*, and connect the definition to the scenario of the transformation being *made up of* line segments, figures, and target shapes.

A term that is used in reference to the position of geometric shapes is *orientation*. Note that *orientation* is a multiple-meaning term. Students may be familiar with a common use of *orientation*, which describes *a meeting or activity used to familiarize people within a new situation*. Define *orientation* as *the relative physical position or direction of something*. In the context of the geometric shapes that are mentioned in Question 1 in the section, *orientation* refers to the *placement* of the shapes on the patty paper.

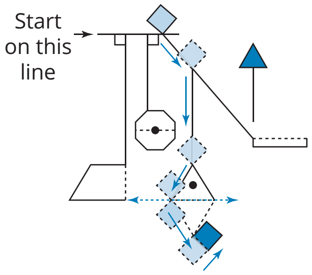
Answers

2. The triangle must start as a horizontal reflection. The diagonal solid line segment carries the translation. The triangle must move down on the rectangle, and then when the rectangle is reflected across the dashed line, the triangle is in position to be translated up.





The square can be translated from its original position to a side of the triangle. A reflection and then translations can move it into the final position.



3. Yes. The images are congruent to the pre-images because they use rigid motion transformations.

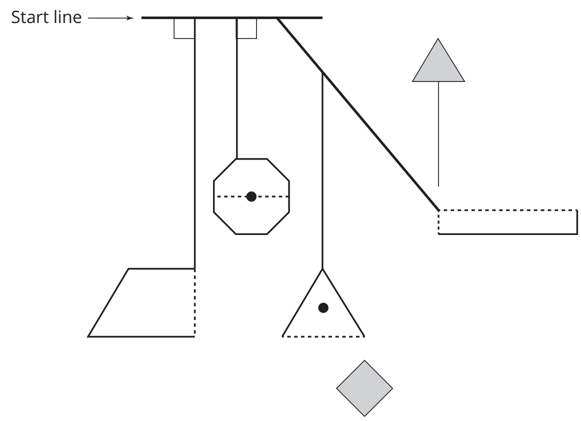
NOTES

Think about:

You can use this diagram to help you predict the effects of your transformations before you test them on the larger diagram at the end of the lesson.

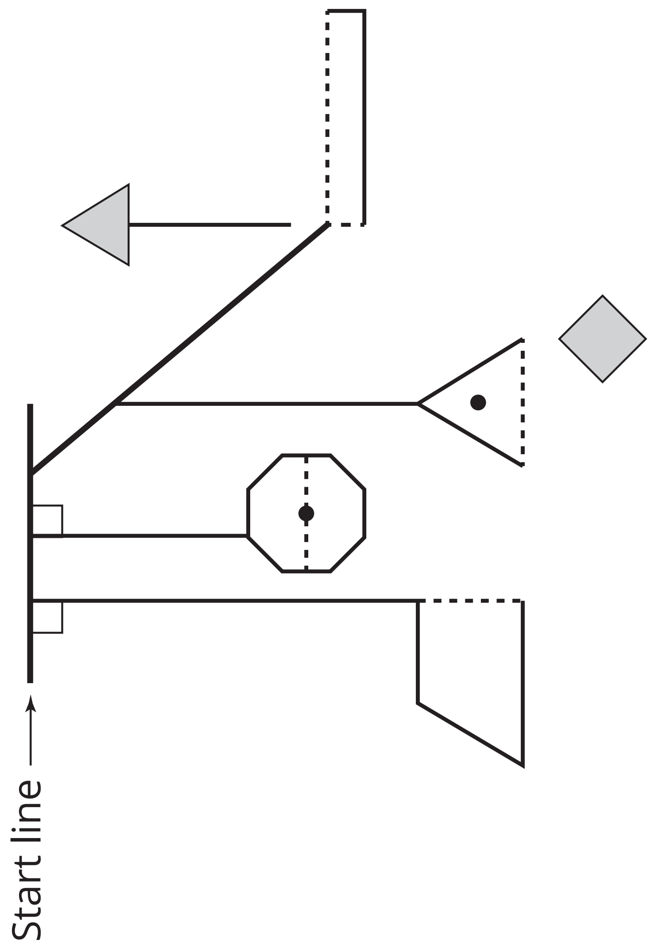
Remember:

A pre-image is a figure prior to a transformation. The image is the figure after the transformation.



2. Describe the sequence of transformations you used to transform each pre-image to each image. Label points on the transformation machine so that you can precisely describe your transformations.

3. Consider the transformations performed on each pre-image to map it onto the image. Are the images congruent to the pre-images? Explain why or why not.


Start line