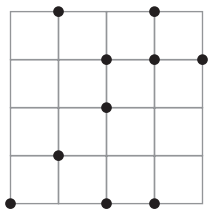
1

Put Your Input In, Take Your Output Out

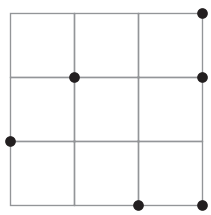
Geometric Components of Rigid Motions

Warm Up

1. Identify the vertices that form a rectangle. Explain why the figure is a rectangle.



2. Identify four vertices that form a trapezoid. Explain why the figure is a trapezoid.



Learning Goals

- Know precise definitions of line segment, angle, and distance along a line.
- Translate lines to produce parallel lines.

Key Terms

- collinear points
- angle
- ray
- rotation angle

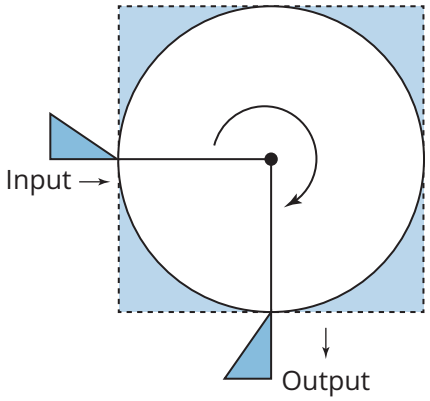
You know a lot about rigid motions, such as translations, reflections, and rotations. How do you use straight lines and angles to represent rigid motion transformations?

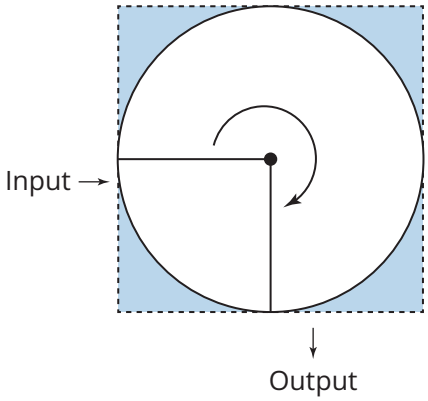
Transformation Machine

You have learned about function machines, which take a value as input and output another value. In this topic, you will learn to think about geometric rigid motions as functions. These functions can be represented with function machines as well, or transformation machines.

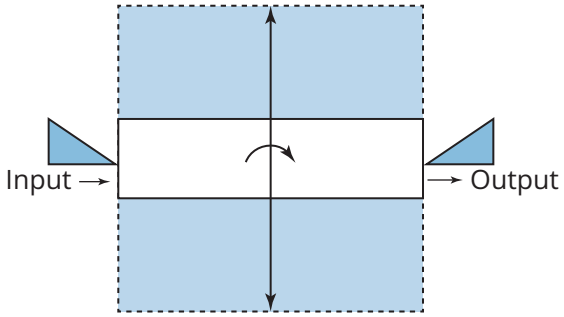
1. Each transformation machine on the left shows a different rigid motion. For each transformation machine on the right, draw an input shape on patty paper. Then apply the transformation and draw the output shape.

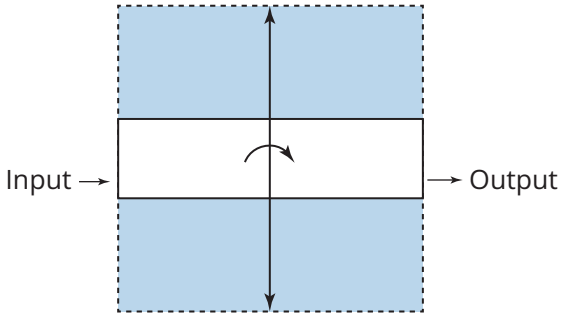
a.



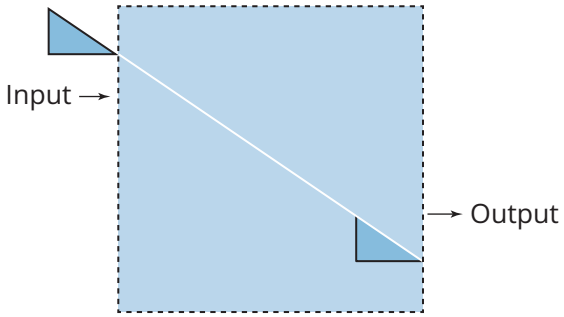


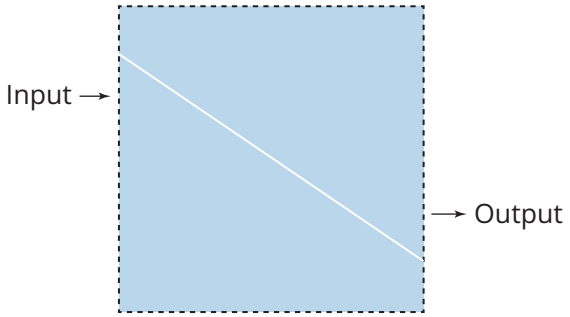
b.





c.





2. **Identify the rigid motion represented by each transformation machine.**

3. **Describe each transformation function. Explain how each input shape is "carried" by geometric objects in the transformation machine to result in the output shape.**

ACTIVITY
1.1 Lines, Line Segments, and Angles

Remember:

A plane extends infinitely in all directions in two dimensions and has no thickness.

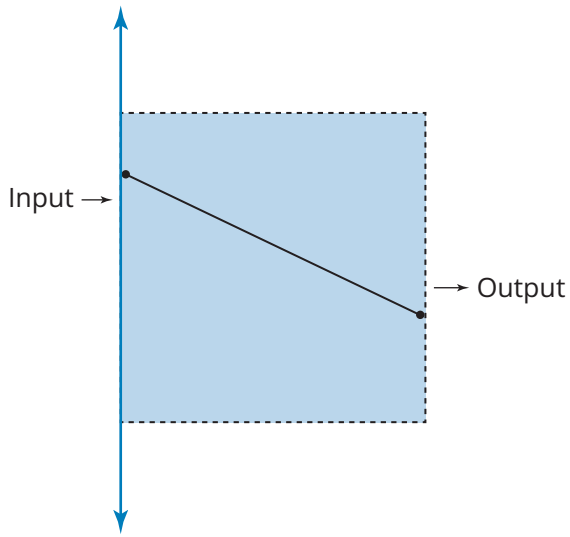
Points and lines are essential building blocks of geometry. They are called undefined terms.

Recall that a point in geometry has no size or shape, but it is often represented using a dot. In a diagram, a point can be labeled using a capital letter. A line is described as a straight, continuous arrangement of an infinite number of points. A line has an infinite length, but no width. Arrowheads are used to indicate that a line extends infinitely in opposite directions. In a diagram, a line can be labeled with a lowercase letter positioned next to the arrowhead.

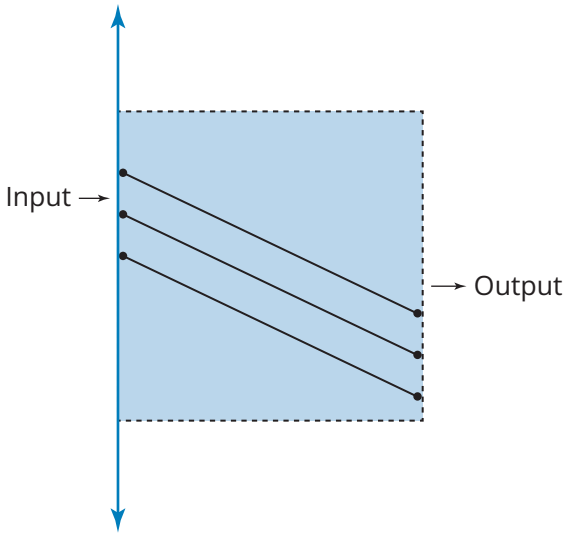
Points that lie along the same line are called **collinear points**. Recall that a line segment is a part of a line between two points on the line, called the endpoints. A distance along a line is the length of a line segment connecting two points on the line. A line segment *AB* has the distance *AB*.

1. Consider the translation machine from the Getting Started.

a. Suppose that the input to the machine is a line as shown. Describe the output of the translation machine.



b. Suppose that the translation machine is a set of parallel line segments as shown. How does this change the output of the machine when the input is a line?



2. Identify the line segments and distances that were used in the translation machines in the previous activity.

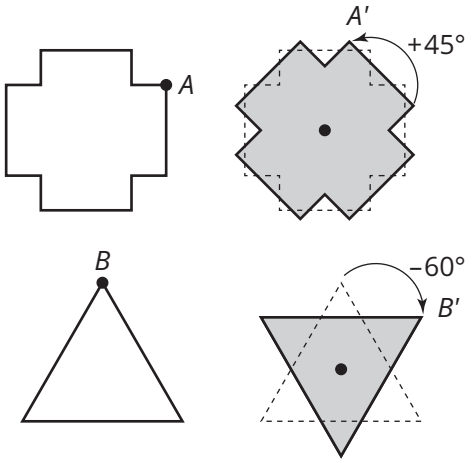
3. Are the line segments in the translation machine in Question 1, part (b), congruent? Use patty paper to justify your answers, and explain your reasoning.

Remember:

Congruent line segments are line segments that have the same length. They represent equal distances.

An **angle** is a set of points consisting of a vertex point and two rays extending from the vertex point. A **ray** is a portion of a line that begins with a single point and extends infinitely in one direction.

A **rotation angle** is a directed angle based on a circle. Positive rotation angles turn counterclockwise, and negative rotation angles turn clockwise.



4. Identify the rotation angle that was used in the rotation machine in the Getting Started.

NOTES

TALK the TALK

Shake It All About

The diagram on the next page shows a transformation machine. The transformation machine is designed to provide a path for each of the input shapes, a triangle and a square, to move from the start line through the machine and map back onto itself. The transformation is comprised of line segments, figures with and without center points, and two target shapes. To use the transformation machine you must first trace the target shape onto patty paper.

The transformation machine has these rules:

- The elements in the transformation machine provide ways to move your input shape. Your input shape must be connected to any of the lines or figures in the transformation machine by at least one vertex in order for it to be translated, rotated, or reflected.
- Any dashed or solid line allows you to translate your input shape.
- Any figure with a solid center point allows you to rotate your input shape around that center. The figure carries your input shape around the rotation.
- Any figure with a dashed line allows you to reflect your input shape across that line. The figure carries your input shape across the reflection.

Use the larger diagram of the transformation machine located at the end of the lesson.

1. **Copy one of the target shapes onto patty paper. Place the input shape on the start line in an orientation of your choosing. Then determine a sequence of translations, reflections, and rotations that maps the input shape onto the corresponding target shape.**

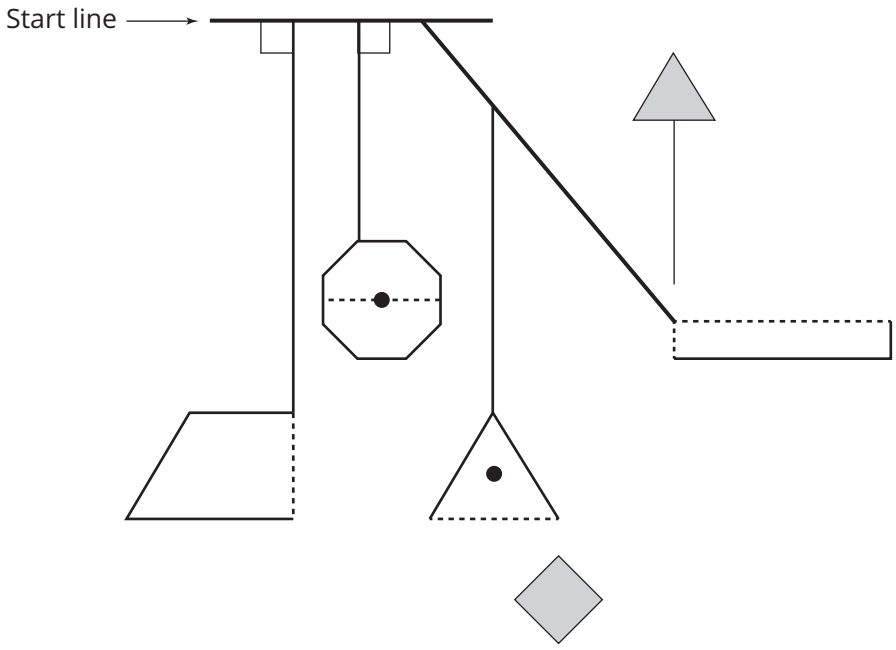
NOTES

Think about:

You can use this diagram to help you predict the effects of your transformations before you test them on the larger diagram at the end of the lesson.

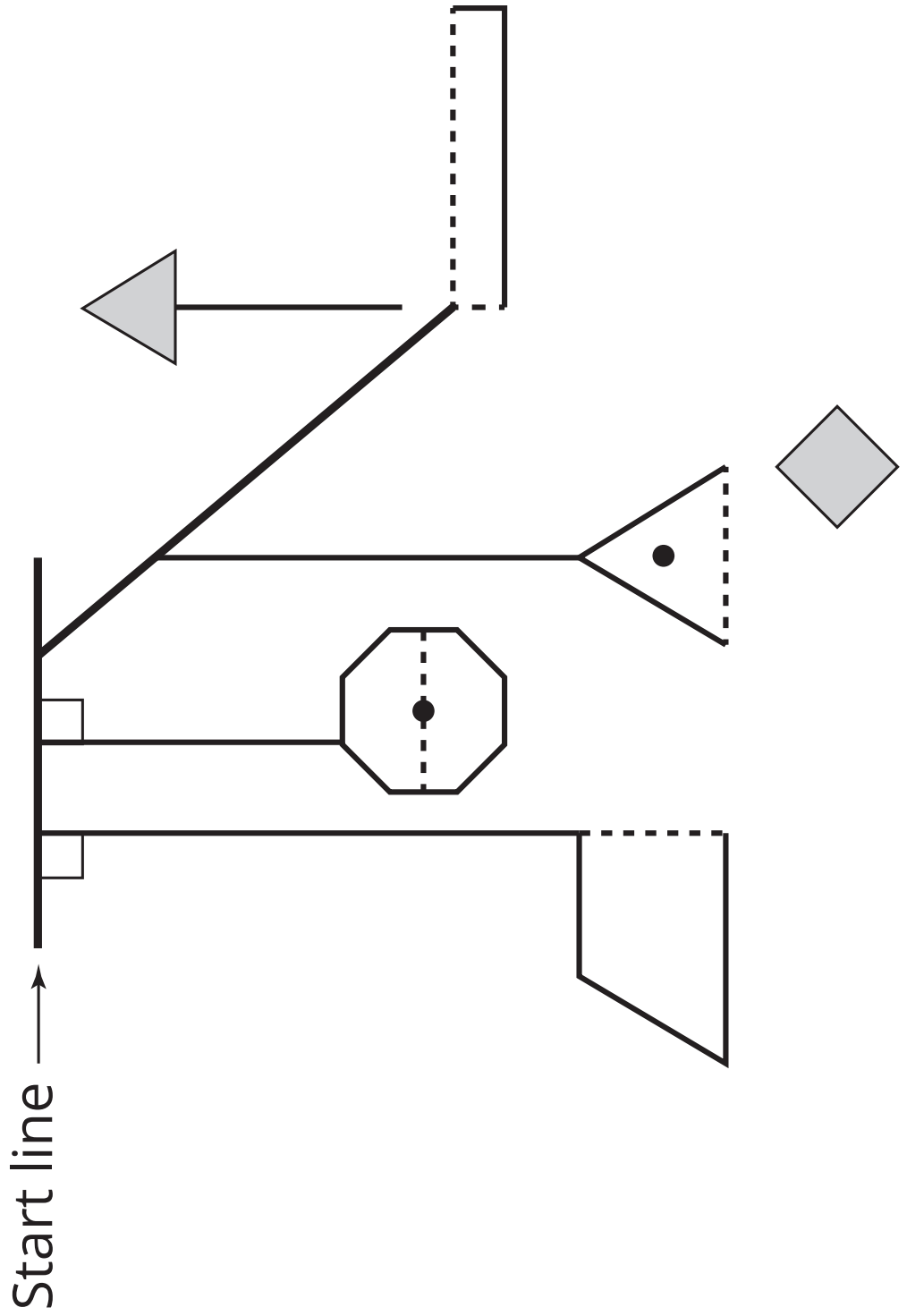
Remember:

A pre-image is a figure prior to a transformation. The image is the figure after the transformation.



2. Describe the sequence of transformations you used to transform each pre-image to each image. Label points on the transformation machine so that you can precisely describe your transformations.

3. Consider the transformations performed on each pre-image to map it onto the image. Are the images congruent to the pre-images? Explain why or why not.


Start line