Lesson 1

Put Your Input In, Take Your Output Out:

Geometric Components of Rigid Motions

Lesson Overview

Students develop the concept that geometric rigid motion transformations can be considered as functions, with rotations, reflections, and translations as the operations. Translations can be described using lines and line segments. Reflections can be described using lines. Rotations can be described using rotation angles. The inputs and outputs are geometric shapes. Each input and its corresponding output have the same size and shape.

Lesson Video(s): The aligned lesson overview video(s) provide additional instruction for students on the key concepts in this lesson and can be found alongside the digital interactive student lesson.

TEKS: G.3B, G.3C

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Transformation Machine

Develop

Activity 1.1: Lines, Line Segments, and Angles

Demonstrate

Talk the Talk: Shake it All About

Getting Started: Transformation Machine

Asynchronous Facilitation Notes

Note: Students will need patty paper, plain paper, or tracing paper for this activity. In this activity, students are introduced to transformations as functions by representing each of three rigid motions (translation, reflection, and rotation) as a machine that takes a shape as an input and produces a congruent shape as an output. Students will draw an input shape on patty paper, perform the transformation, and then use the free draw feature to draw the input shape and output shape to represent each rigid motion. Students identify the rigid motion represented by each transformation machine and describe each transformation function in additional free response questions.

If students do not have access to patty paper, some suggested replacements for patty paper are: paper from shoeboxes, wax paper from the kitchen, and holding regular paper up to a window or backlit screen.

Synchronous Facilitation Notes

In this activity, students are introduced to transformations as functions by representing each of three rigid motions (translation, reflection, and rotation) as a machine that takes a shape as an input and produces a congruent shape as an output. They then draw another input shape and output shape to represent each rigid motion.

Ask a student to read the introduction. Discuss as a class.

Questions to ask

- What is an example of a function machine?
- What is the input of a function machine? What is its output?
- What is meant by a transformation machine?
- What is the input of a transformation machine? What is its output?
- If it is a rigid motion transformation machine, what operations can it perform?

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

To assist all students,

- Discuss the first machine in Question 1 part (a) as a class.
- Have students use patty paper to demonstrate the operations in the left column before creating their own input shape.

As students work, look for

- Use of the terms *rotation*, *reflection*, and *translation*.
- Detailed responses that include direction and degree measure of the rotation, the orientation of the line of reflection, and the horizontal and vertical nature of the translation.
- Different shapes used to model the same transformation machine.

Misconception

Confusion as to why the output in Question 1, part (c) remains inside the box. The translation is a specified distance. When the bottom right vertex of the triangle is translated to the specific distance of the line segment, it remains inside the box, and the other points on the triangle lie behind it, translated the same distance.

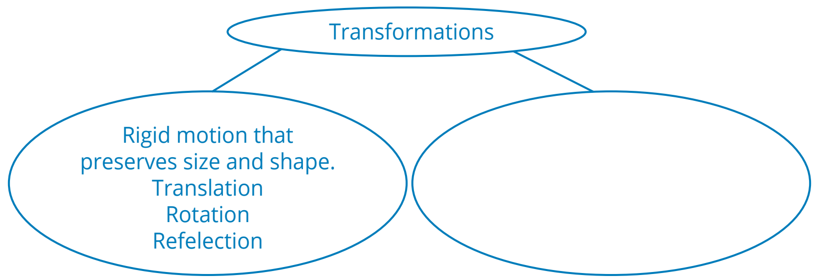
Questions to ask

- What shape did you use?
- What helped you determine that the transformation was a rotation?

- Is the rotation clockwise or counterclockwise? How do you know?
- How many degrees is the figure rotated? How do you know?
- What does the arrow in this transformation machine suggest?
- What does the vertical line in the middle of this machine suggest?
- What does the diagonal line in this transformation machine suggest?
- Do all of these transformation machines preserve size?
- Do all of these transformation machines preserve shape?

Differentiation strategy

To extend the activity, provide a framework to summarize the vocabulary used in this lesson and contrast rigid motions from dilations.



Summary

Geometric rigid motion transformations can be considered as functions, with rotations, reflections, and translations as the operations. The inputs and outputs are geometric shapes. Each input and its corresponding output have the same size and shape.

Activity 1.1:
Lines, Line Segments, and Angles

Asynchronous Facilitation Notes

In this activity, students further investigate function machines in free response questions. Students either review, or are introduced to, terms necessary to experiment with transformations in the plane. They also address both positive and negative angles of rotation. Note that students will need patty paper, tracing paper, or plain paper to complete this activity.

Synchronous Facilitation Notes

In this activity, students either review or are introduced to terms necessary to experiment with transformations in the plane. They also address both positive and negative angles of rotation.

Ask a student to read the introduction and definitions aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- Does it matter where the arrows are placed on the translated line? Why or why not?
- Why doesn't the output change?
- How would this question be different if a point was labeled on the line?
- Demonstrate the input and output if a point was placed on the line.

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

Questions to ask

- Is a point an actual dot? How is a point named?
- Does a line have length and width? How is a line named?
- How many line segments are contained on a line?
- How are a line and a line segment different?
- What do a line and a line segment have in common?
- Are two points always collinear? Why or why not?
- How can patty paper be used to show that the line segments in Question 1, part (b) are congruent.

Misconception

Students may not understand the difference between describing and defining a geometric term. A discussion about undefined terms and defined terms may be helpful. The three undefined terms in geometry are *point*, *line*, and *plane*. These undefined terms are the building blocks of geometry and they do not have dimension. All other terms are defined with an understanding of these basic elements.

Ask a student to read the definitions before Question 4 aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

Misconception

It may not be intuitive that positive rotation angles turn counterclockwise and negative rotation angles turn clockwise. This concept aligns with angle placement on the coordinate plane. A positive rotation angle with one ray at 0° opens up from the x-axis towards the y-axis in the first quadrant.

Questions to ask

- What do a ray and a line have in common?
- How is a ray different from a line?
- Why do you think a rotation angle is based on a circle?

Differentiation strategies

To extend the activity,

- Have students use a protractor to verify that the measures of the angles in the diagrams are 45° and 60°.
- Have students investigate what a 45° angle and a 60° angle look like on the coordinate plane.

- What benchmark angle(s) did you use to identify the 270° angle?
- How do you know whether the angle measure is positive or negative?

Summary

Translations can be described using lines and line segments. Reflections can be described using lines. Rotations can be described using rotation angles.

Talk the Talk: Shake It All About

Asynchronous Facilitation Notes

Note: Students will need patty paper, tracing paper, or plain paper to complete this activity. In this activity, students are given a transformation machine composed of line segments, figures with and without center points, and two target shapes. Students watch a video showing how to use the transformation machine. Then students trace one of the two target shapes onto patty paper. They then determine a path for each of the input shapes, a triangle and a square, to move from the start line through the machine and map back onto itself. Students then describe the sequences of transformations they used in free response questions. The free draw feature is used to label points on the transformation machine allowing students to precisely describe their transformations. Students also explain why each image is congruent to its pre-image in an additional free response question.

Synchronous Facilitation Notes

In this activity, students are given a transformation machine composed of line segments, figures with and without center points, and two target shapes. Students watch a video showing how to use the transformation machine. To use the transformation machine students must first trace one of the two target shapes onto patty paper. They then determine a path for each of the input shapes, a triangle and a square, to move from the start line through the machine and map back onto itself. Students then describe the sequences of transformations they used.

Ask a student to read the introduction and rules aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

To scaffold support,

- Discuss the components of the transformation machine.
- Complete an example as a class.

Questions to ask regarding the components of the transformation machine

- What does a vertical translation look like on this machine?
- What does a vertical reflection look like on this machine?
- What does a horizontal reflection look like on this machine?

- What transformation does the dotted line on the trapezoid suggest?
- Does the dotted line on the trapezoid suggest a vertical or horizontal reflection?
- What transformation does the dotted line on the octagon suggest?
- Does the dotted line on the octagon suggest a vertical or horizontal reflection?
- What transformation does the dotted line on the triangle suggest?
- Does the dotted line on the triangle suggest a vertical or horizontal reflection?
- What transformation does the dotted line on the rectangle suggest?
- Does the dotted line on the rectangle suggest a vertical or horizontal reflection?
- How many locations are available on the start line?

Misconception

Students may not understand that a reflection carries the entire image across the line of reflection. The original figure can be traced across the line of reflection, and its sides can be used for translations.

Questions to ask regarding a transformation of the triangle

- Which of the three possible start locations can lead your shape to the triangle?
- If you place your patty paper image of the triangle on the third (last) start position and slide it down the diagonal guideline, what transformation must it undergo when it moves through the rectangle?
- If the rectangle causes a horizontal reflection, what should be the orientation of the triangle before it undergoes this transformation?
- If the pre-image of the triangle is initially oriented on the start line as a horizontal reflection of the image, and then slides down the diagonal guideline, and the rectangle reflects it across the dotted line, will the triangle be in the correct position to be translated up to the position of the image?

Questions to ask regarding a transformation of the square

- Which of the three possible start locations can lead your shape to the square?
- If you place your patty paper image of the square on the third (last) start position and slide it down the diagonal guideline, and then slide it down the vertical guideline, can you position the side of the square to line up with the side of the triangle?
- What transformation must it undergo when it moves through the triangle?
- If the triangle causes a horizontal reflection, what should be the orientation of the square before it undergoes this transformation?
- If the pre-image of the square is initially oriented on the start line the same way the image is oriented, and then slides down the diagonal, then vertical guideline, and the triangle reflects it across the dotted line, will the square be in the correct position to be translated up to the position of the image?

Summary

A combination of rigid motion transformations can map a figure back onto itself.