1

Pi: The Ultimate Ratio

Exploring the Ratio of Circle Circumference to Diameter

MATERIALS

Centimeter ruler
String
Compass
Calculator with π key

Lesson Overview

Students explore the relationship between the distance around a circle and the distance across a circle. They learn the terms *circumference, diameter, and radius*. Students use hands-on tools to measure the distance around a circle and the length of its diameter. They write a ratio of the circle's circumference to its diameter, and they compare their data to the data of their peers. They then use a compass to create their own circles and realize that for every circle the ratio of circumference to diameter is pi. Students practice solving for the diameter or the circumference in problems.

Grade 7

Proportionality

(5) The student applies mathematical process standards to use geometry to describe or solve problems involving proportional relationships. The student is expected to:

(B) describe π as the ratio of the circumference of a circle to its diameter.

Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to develop geometric relationships with volume. The student is expected to:

(C) use models to determine the approximate formulas for the circumference and area of a a circle and connect the models to the actual formulas.

(9) The student applies mathematical process standards to solve geometric problems. The student is expected to:

(B) determine the circumference and area of circles.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- The circumference of a circle is the distance around the circle.
- The ratio of the circumference of a circle to the diameter of a circle is approximately 3.14 or pi.

- The formula for calculating the circumference of a circle is $C = d\pi$ or $C = 2\pi r$ where C is the circumference of a circle, d is the length of the diameter of the circle, r is the length of the radius of the circle, and π is represented using the approximation 3.14.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Across and Around

Students use a string and a ruler to measure the distance around a circle. They then measure from a point on the circle to the center. This activity is designed to engage students in thinking about circumference and radius as lengths that are constant properties of any given circle.

Develop

Activity 1.1: Analyzing the Parts of a Circle

Students match the vocabulary words *radius*, *diameter*, and *circumference* to the parts of the circle they have been analyzing in the previous activities.

Activity 1.2: Measuring the Distance Around a Circle

Students again use string and a ruler to measure the distances across (diameter) and around (circumference) five different circles, one of which is congruent to the circle from the Getting Started activity. Students notice that the ratio of the distance around to the distance across each circle is the same for all circles. The approximate value of this ratio (pi) is about 3.14.

Day 2

Activity 1.3: The Circumference Formula

Students use what they have discovered in the previous activities to write a formula for the circumference of any circle. They then use this formula to compute circumferences given diameters and diameters given circumferences.

Demonstrate

Talk the Talk: Twice

Students compare the radii, diameters, and circumferences of two circles, one with a radius twice as long as the other. They are guided to notice that a circle with a radius half as long has a circumference half as long as well, while the ratio of circumference to diameter (pi) remains constant.

Getting Started: Across and Around

ENGAGE

Facilitation Notes

In this activity, using a string and centimeter ruler, students measure the distance around a circle (circumference) and the distance from the center point to any point on the circle (radius). They compare their results with classmates to conclude that this ratio is a constant.

Have students work with a partner or in groups to complete Question 1. Share responses as a class.

Questions to ask

- Are all circles the same shape?
- Are all circles the same size?
- Explain how you used your string and ruler to determine the distance around the circle.
- Why did you need string? Would a measuring tape have worked?
- Should there be more than one correct answer? Why or why not?
- Would you expect everyone to get the same results? Why or why not?
- What error would account for different results?
- What could be a reason your classmate gets an answer different than yours?
- What word or term describes the distance around a circle?

Have students complete Question 2 and then share responses as a class.

As students work, look for

- Incorrect units of measure.
- Measurement errors.

Questions to ask

- Did everyone choose the same point on the circle?
- What word or term describes the distance from the center point to any point on the circle?
- Why is everyone getting the same results?

Summary

The distance from the center of a circle to any point on the circle is the same.

DEVELOP

Activity 1.1
Analyzing the Parts of a Circle

Facilitation Notes

In this activity, the terms *circle*, *radius*, *diameter* and *circumference* are formally introduced and students use these terms to answer related questions.

Ask a student to read the introduction aloud. Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Are all of the points you have drawn considered on the circle or in the circle?
- What is the difference between a point located on a circle and a point located in a circle?
- Are circles considered polygons? Why or why not?
- What are some familiar objects in your house that are shaped like circles?
- What are some familiar objects in your house that are shaped like spheres?
- What shape is an inflated basketball?
- What shape is a deflated basketball?
- How are circumference and perimeter related?

Ask a student to read the definition of a circle aloud and answer Question 4 part (a).

Questions to ask

- Can a circle have more than one center point? Explain.
- How does this figure look like the figure you drew in Question 1?
- How many points are on a circle?

Ask a student to read through the next paragraph and answer parts (b) through (d) as a class.

Questions to ask

- How many radii are in a circle?
- How do the lengths of each radius in a circle compare?
- How many diameters are in a circle?
- How do the lengths of each diameter in a circle compare?
- How are radii different from diameters?
- How are radii similar to diameters?

Misconception

- Students sometimes think that there is only one diameter of a circle, and it is a horizontal line segment through the center of the circle. Discuss the error in that thinking as you answer Question 4 part (c).
- Students sometimes think a circle includes the points in its interior. Use the definition to clarify their thinking.

Have students work with a partner in a group to complete Question 5. Share responses as a class.

Questions to ask

- What relationship exists between the radius and the diameter of a circle?
- Is there another way to describe this relationship?

Summary

All points on a circle are equidistant from the center point. All radii of the same circle are the same length, or congruent. The length of the radius of a circle is half the length of the diameter of a circle. The length of the diameter of a circle is twice the length of a radius of a circle.

Activity 1.2

Measuring the Distance Around a Circle

Facilitation Notes

In this activity, using a string and centimeter ruler, students measure the diameters and circumferences of various circles. The underpinnings for pi are laid as students identify the ratio of the distance around each circle to the distance across each circle. They calculate the approximate value of this ratio (pi) to be about 3.14. The goal of this activity is for students to investigate the ratio between the circumference of any circle to its diameter, not to practice division. Allow students to use a calculator to average the ratios recorded.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Misconception

Students sometimes use the terms *equal* and *congruent* interchangeably. The term *equal* is used for numbers. The term *congruent* is used for shapes.

As students work, look for

- Ratios with the numerator and denominator reversed.
- Errors in computing the average of the ratios.

Questions to ask

- Were any of the circles more difficult to measure than the others? Why or why not?
- Can you think of a better way to determine circumference than using string and a ruler?
- Do you see any patterns in your table?
- Is there a way to get a more exact answer when using the string and ruler?
- Does the average of the answers in the last column look familiar?
- Do you recognize this ratio?
- Would you expect this ratio to be the same if the circumferences are not the same?
- Should this ratio be the same for all sizes of circles? Why or why not?

Summary

The ratio of the circumference to the diameter of any circle is approximately 3.14.

Activity 1.3
The Circumference Formula

Facilitation Notes

In this activity, students create a formula for the circumference of any circle and use it to compute unknown values.

Have students work with a partner in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

For scaffold support for students as they rewrite the formula, help them make the connection to fact families.

Questions to ask

- Is π a rational number? What is the definition of a rational number?
- What is the relationship between the diameter and the radius?
- Does $d = 2r$ or does $r = 2d$?
- What is the relationship between the formula $C = \pi d$ and $C = 2\pi r$?

- If the radius is known, how can you determine the diameter?
- If the diameter is known, how can you determine the radius?
- If the radius is known, how can you determine the circumference?
- If the diameter is known, how can you determine the circumference?
- If the circumference is known, how can you determine the diameter?
- If the circumference is known, how can you determine the radius?

Have students work with a partner to complete Question 3. Share responses as a class.

Questions to ask

- Which answer is exact?
- How do your calculations using 3.14 compare to those calculations using the π key on a calculator?
- How do your calculations using $\frac{22}{7}$ compare to those calculations using the π key on a calculator?

Have students work with a partner to complete Questions 4 and 5. Share responses as a class.

Summary

The formula $C = \pi d$ and $C = 2\pi r$ can be used to calculate the circumference of a circle where d is the diameter and r is the radius.

Talk the Talk: Twice

DEMONSTRATE

Facilitation Notes

In this activity, students determine the lengths of the radius, diameter, and circumference of two different circles. They conclude the ratio of circumference to diameter (pi) remains constant in both situations.

Have students work with a partner in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- If the length of the radius is equal to 3 cm, what is the length of the diameter?
- If the length of the diameter is equal to 3 cm, what is the length of the radius?

- Would you expect the circumference of the circle in Question 1 part (b) to be half the circumference of the circle in Question 1 part (a)? Why or why not?
- Will circumference to diameter ratio be approximately 3.14 in any situation?

Differentiation strategy

To extend the activity, have students draw the two circles on graph paper. Then, have them investigate the relationship between the areas of the two circles. The areas of the circles can be estimated by counting square units.

Summary

The ratio of the circumference of a circle to its diameter is pi (π). You can use 3.14 or $\frac{22}{7}$ as approximations for π.

1 Pi: The Ultimate Ratio

Exploring the Ratio of Circle Circumference to Diameter

WARM UP

Scale up or down to determine an equivalent ratio.

1. $\frac{18 \text{ miles}}{3 \text{ hours}} = \frac{?}{1 \text{ hour}}$
2. $\frac{\$750}{4 \text{ days}} = \frac{?}{1 \text{ day}}$
3. $\frac{12 \text{ in.}}{1 \text{ ft}} = \frac{?}{5 \text{ ft}}$
4. $\frac{48 \text{ oz}}{3 \text{ lb}} = \frac{?}{1 \text{ lb}}$

LEARNING GOALS

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Understand the formula for the circumference of a circle, and use the formula to solve problems.

KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi

You have learned about ratios. How can you use ratios to analyze the properties of geometric figures, such as circles?

Warm Up Answers

1. 6 miles
2. $187.50
3. 60 in.
4. 16 oz

Answers

Answers are approximates. Sample answers.

1a. Everyone's measurements are close to 14.4 cm.

1b. Everyone's measurements should be the same, but it is difficult to be exact measuring with string.

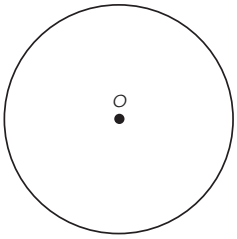
2a. Everyone's measurements are close to 2.3 cm.

2b. Any segment from a point on the circle to the center is the same length.

Getting Started

Across and Around

A circle is shown with a point drawn at the center of the circle. The name of the point is *O*, so let's call this Circle *O*.



1. Analyze the distance around the circle.

a. Use a string and a centimeter ruler to determine the distance around the circle.

b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

2. Draw a line from a point on the circle to the center of the circle, point *O*.

a. Measure your line using your centimeter ruler.

b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

ACTIVITY
1.1

Analyzing the Parts of a Circle

Everyone can identify a circle when they see it, but defining a circle is a bit harder. Can you define a circle without using the word *round*? Investigating how a circle is formed will help you mathematically define a circle.

1. Follow the given steps to investigate how a circle is formed.

Step 1: In the space provided, draw a point and label the point *A*.

Step 2: Use a centimeter ruler to locate and draw a second point that is exactly 5 cm from point *A*. Label this point *B*.

Step 3: Locate a third point that is exactly 5 cm from point *A*. Label this point *C*.

Step 4: Repeat this process until you have drawn at least ten distinct points that are each exactly 5 cm from point *A*.

2. How many other points could be located exactly 5 cm from point *A*? How would you describe this collection of points in relation to point *A*?

3. Define the term *circle* without using the word *round*.

Answers

1. Check students' circles.
2. An infinite number of points could be located exactly 5 cm from point *A*.
3. A circle is a collection of points on the same plane equidistant from the same point.

Answers

4a. The circle shown is Circle *B*. Point *B* is the center of the circle.

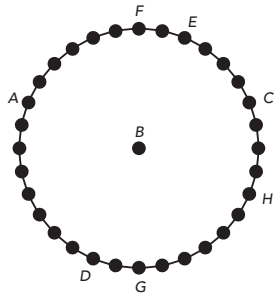
4b. Sample answer. Line segment *AB* is a radius of Circle *B*.

4c. Sample answer. Line segment *FG* is a diameter of Circle *B*.

4d. All radii of the same circle must be the same length because all of the points on the circle are equidistant from the center point.

5. The length of a radius is half the length of a diameter because two radii form each diameter.

A **circle** is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant. Circles are named by their center point.

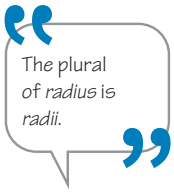


4. Use the circle shown to answer each question.

a. Name the circle.

The **radius** of a circle is a line segment formed by connecting a point on the circle and the center of the circle. The distance across a circle through the center is the diameter of the circle. The **diameter** of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point. The distance around a circle is called the **circumference** of the circle.

b. Identify a radius of the circle.



c. Identify a diameter of the circle.

d. Are all radii of this circle the same length? Explain your reasoning.

5. What is the relationship between the length of a radius and the length of a diameter?

ACTIVITY
1.2

Measuring the Distance Around a Circle

Let's explore circles. Use circles *A*, *B*, *D*, *E*, and *O* provided at the end of the lesson. Circle *O* is the same as the circle from the activity *Across and Around*.

1. **Use a string and a centimeter ruler to measure the distance from a point on the circle to the center and the distance around each circle. Record your measurements in the table. In the last column, write the ratio of *Circumference* : *Diameter* in fractional form.**

Circle	Circumference	Radius	Diameter	$\frac{\text{Circumference}}{\text{Diameter}}$
Circle *A*				
Circle *B*				
Circle *O*				
Circle *D*				
Circle *E*				

2. **Average the ratios recorded for $\frac{\text{Circumference}}{\text{Diameter}}$. What is the approximate ratio for the circumference to the diameter for the set of circles? Write the approximate ratio as a fraction and as a decimal.**

3. **How does your answer to Question 2 compare to your classmates' answers?**

4. **Average all of your classmates' answers to Question 2. Write the approximate ratio of circumference to the diameter as a fraction and as a decimal.**

Answers

1. See table below.
2. See table. Answers are approximations.
3. For each circle, everyone's answers were the same or close to each other.
4. Answers will vary. The average should be close to 3.14.

1.

Circle	Circumference	Radius	Diameter	Circumference / Diameter
Circle *A*	7.5 cm	1.2 cm	2.4 cm	7.5/2.4 = 3.13
Circle *B*	11.9 cm	1.9 cm	3.8 cm	11.9/3.8 = 3.13
Circle *O*	14.4 cm	2.3 cm	4.6 cm	14.4/4.6 = 3.13
Circle *D*	20.1 cm	3.2 cm	6.4 cm	20.1/6.4 = 3.14
Circle *E*	23.9 cm	3.8 cm	7.6 cm	23.9/7.6 = 3.14

Answers

1. $C = \pi d$
2. $C = 2\pi r$

3a. See table below.

NOTES

ACTIVITY 1.3

The Circumference Formula

The number **pi** (π) is the ratio of the circumference of a circle to its diameter. That is pi $= \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi = \frac{C}{d}$, where C is the circumference of the circle, and d is the diameter of the circle. The number π has an infinite number of decimal digits that never repeat. Some approximations used for the value π are 3.14 and $\frac{22}{7}$.

1. **Use this information to write a formula for the circumference of a circle, where *d* represents the diameter of a circle and *C* represents the circumference of a circle.**

2. **Rewrite the formula for the circumference of a circle, where *r* represents the radius of a circle and *C* represents the circumference of a circle.**

3. **Use different representations for π to calculate the circumference of a circle.**

   a. **Calculate the circumference of a circle with a diameter of 4.5 centimeters and a circle with a radius of 6 inches. Round your answer to the nearest ten-thousandths, if necessary.**

Value for π	$d = 4.5$ centimeters	$r = 6$ inches
π		
Use the π key on a calculator		
Use 3.14 for π		
Use $\frac{22}{7}$ for π		

3a.

Value for π	$d = 4.5$ centimeters	$r = 6$ inches
π	4.5π cm	12π in.
Use the π key on a calculator	$\approx$ 14.1372 cm	$\approx$ 37.6991 in.
Use 3.14 for π	$\approx$ 14.13 cm	$\approx$ 37.68 in.
Use 22/7 for π	$\approx$ 14.1429 cm	$\approx$ 37.7143 in.

b. Compare your circumference calculations. How do the different values of π affect your calculations?

4. Use the circumference of a circle formula to determine each unknown. Use 3.14 for π.

a. Compute the diameter of the circle with a circumference of 65.94 feet.

b. Compute the radius of the circle with a circumference of 109.9 millimeters.

5. What is the minimum amount of information needed to compute the circumference of a circle?

Answers

3b. The calculations that leave π in the answer represent the exact circumference. The calculations using the π key on the calculator are the best circumference approximations. The calculations using 3.14 for π are slightly less than those approximations using the π key. The calculations using $\frac{22}{7}$ for π are slightly greater than those approximations using the π key.

4a. $d \approx 21$ ft

4b. $r \approx 17.5$ mm

5. The length of the radius or the length of the diameter must be known.

Answers

1a. Check students' drawings.

1b. Check students' drawings.

2. The circles in Question 1 both have the same ratio of circumference to diameter.

   The circle in Question 1b has a radius that is half as long as the circle in Question 1a.

3. The circumference of the circle in Question 1b is half the circumference of the circle in Question 1a.

4. All circles have the same circumference-to-diameter ratio, which is π.

NOTES

TALK the TALK

Twice

Use what you have learned to compare circles by their characteristics.

1. Using your compass, draw each circle.

a. radius length of 3 centimeters

b. diameter length of 3 centimeters

2. Describe the similarities and differences between your two circles.

3. Describe the relationship between the circumferences of the two circles.

4. Describe the circumference-to-diameter ratio of all circles.

Measuring the Distance Around a Circle