Lesson 1

Pi: The Ultimate Ratio:

Exploring the Ratio of Circle Circumference to Diameter

Lesson Overview

Students explore the relationship between the distance around a circle and the distance across a circle. They learn the terms *circumference, diameter,* and *radius*. Students use tools to measure the distances and compare the ratio of the circumference to the length of the diameter. They then examine different circle measurements and realize that for every circle the ratio of circumference to diameter is pi. Students practice solving for the diameter or the circumference in problems.

Lesson Video(s): The aligned lesson overview video(s) provide additional instruction for students on the key concepts in this lesson and can be found alongside the digital interactive student lesson.

TEKS: 7.5B, 7.8C, 7.9B

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Across and Around

Develop

Activity 1.1: Analyzing the Parts of a Circle
Activity 1.2: Measuring the Distance Around a Circle

Day 2

Activity 1.3: The Circumference Formula

Demonstrate

Talk the Talk: Twice

Getting Started: Across and Around

Asynchronous Facilitation Notes

In this activity, students determine the best way to measure the distance around a circle using a ruler and some string.

Synchronous Facilitation Notes

In this activity, students determine the best way to measure the distance around a circle using a ruler and some string.

Have students work with a partner or in groups to complete the activity. Share responses as a class.

Questions to ask

- Are all circles the same shape?
- Are all circles the same size?
- Why did you need a string? Would a measuring tape have worked?
- If we actually measured this circle with those materials, should there be more than one correct answer? Why or why not?
- Would you expect everyone to get the same results? Why or why not?
- What error would account for different results?
- What could be a reason your classmate gets an answer different than yours?
- What word or term describes the distance around a circle?

Summary

The distance from the center of a circle to any point on the circle is the same.

Activity 1.1:
Analyzing the Parts of a Circle

Asynchronous Facilitation Notes

In this activity, the terms *circle, radius, diameter* and *circumference* are formally introduced and students use these terms to answer related questions. Remind students to use the metric ruler tool. They can rotate the ruler tool and start measuring from any tick mark as long as the distance from the original point is 5 cm.

Synchronous Facilitation Notes

In this activity, the terms *circle, radius, diameter* and *circumference* are formally introduced and students use these terms to answer related questions.

Ask a student to read the introduction aloud. Have students work with a partner or in a group to complete Questions 1 through 3. Remind students that they can rotate the metric ruler tool and start measuring from any tick mark as long as the distance from the original point is 5 cm. Share responses as a class.

Questions to ask

- What is the difference between a point located on a circle and a point located in a circle?
- Are circles considered polygons? Why or why not?

- What are some familiar objects in your house that are shaped like circles?
- What are some familiar objects in your house that are shaped like spheres?
- What shape is an inflated basketball?
- What shape is a deflated basketball?

Ask a student to read the definition of a circle aloud.

Questions to ask

- Can a circle have more than one center point? Explain.
- How many points are on a circle?

Ask a student to read the definitions of *radius*, *diameter*, and *circumference*. Complete Question 4 as a class.

Questions to ask

- How many radii are in a circle?
- How do the lengths of each radius in a circle compare?
- How many diameters are in a circle?
- How do the lengths of each diameter in a circle compare?
- How are radii different from diameters?
- How are radii similar to diameters?
- How are circumference and perimeter related?

Misconception

- Students sometimes think that there is only one diameter of a circle, and it is a horizontal line segment through the center of the circle. Discuss the error in that thinking as you share responses when students identify a diameter.
- Students sometimes think a circle includes the points in its interior. Use the definition to clarify their thinking.

Have students work with a partner in a group to complete Question 5. Share responses as a class.

Questions to ask

- What relationship exists between the radius and the diameter of a circle?
- Is there another way to describe this relationship?

Summary

All points on a circle are equidistant from the center point. All radii of the same circle are the same length, or congruent. The length of the radius of a circle is half the length of the diameter of a circle. The length of the diameter of a circle is twice the length of a radius of a circle.

Activity 1.2:
Measuring the Distance Around a Circle

Asynchronous Facilitation Notes

In this activity, students see a variety of circles and the corresponding measurements for each circle's radius, diameter and circumference. Students compare the information in the table and find the ratio of circumference to diameter for each circle. Students will represent the ratio as both a fraction and a decimal. The underpinnings for pi are laid as students identify the ratio of the distance around each circle to the distance across each circle. They calculate the approximate value of this ratio (pi) to be about 3.14. The goal of this activity is for students to investigate the ratio between the circumference of any circle to its diameter, not to practice division. Allow students to use a calculator to average the ratios recorded.

Synchronous Facilitation Notes

In this activity, students see a variety of circles and the corresponding measurements for each circle's radius, diameter and circumference. Students compare the information in the table and find the ratio of circumference to diameter for each circle. Students will represent the ratio as both a fraction and a decimal. The underpinnings for pi are laid as students identify the ratio of the distance around each circle to the distance across each circle. They calculate the approximate value of this ratio (pi) to be about 3.14. The goal of this activity is for students to investigate the ratio between the circumference of any circle to its diameter, not to practice division. Allow students to use a calculator to average the ratios recorded.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

Misconception

Students sometimes use the terms *equal* and *congruent* interchangeably. The term *equal* is used for numbers. The term *congruent* is used for shapes.

As students work, look for

- Ratios with the numerator and denominator reversed.
- Errors in computing the average of the ratios.

Questions to ask

- Do you see any patterns in the table?
- Does the average of the answers in the last column look familiar?
- Do you recognize this ratio?
- Would you expect this ratio to be the same if the circumferences are not the same?
- Should this ratio be the same for all sizes of circles? Why or why not?

Summary

The ratio of the circumference to the diameter of any circle is approximately 3.14.

Activity 1.3:
The Circumference Formula

Asynchronous Facilitation Notes

In this activity, students create a formula for the circumference of any circle and use it to compute unknown values. Students manipulate this formula and use it with different representations for π to see how different values for π affect calculations. Students will need a calculator for this that contains a π key.

Synchronous Facilitation Notes

In this activity, students create a formula for the circumference of any circle and use it to compute unknown values.

Have students work with a partner in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To scaffold support with rewriting the formula, help them make the connection to fact families.

Questions to ask

- Is π a rational number? What is the definition of a rational number?
- What is the relationship between the diameter and the radius?
- Does $d = 2r$ or does $r = 2d$?
- What is the relationship between the formula $C = \pi d$ and $C = 2\pi r$?
- If the radius is known, how can you determine the diameter?
- If the diameter is known, how can you determine the radius?
- If the radius is known, how can you determine the circumference?
- If the diameter is known, how can you determine the circumference?
- If the circumference is known, how can you determine the diameter?
- If the circumference is known, how can you determine the radius?

Have students work with a partner or a small group to complete Question 3. Share responses as a class.

Questions to ask

- Which answer is exact?
- How do your calculations using 3.14 compare to those calculations using the π key on a calculator?
- How do your calculations using $\frac{22}{7}$ compare to those calculations using the π key on a calculator?

Have students work with a partner to complete Questions 4 and 5. Share responses as a class.

Summary

The formula $C = \pi d$ and $C = 2\pi r$ can be used to calculate the circumference of a circle where d is the diameter and r is the radius.

Talk the Talk: Twice

Asynchronous Facilitation Notes

In this activity, students determine the lengths of the radius, diameter, and circumference of two different circles. They conclude the ratio of circumference to diameter (pi) remains constant in both situations. Students will need to upload an image of their circles.

Synchronous Facilitation Notes

In this activity, students determine the lengths of the radius, diameter, and circumference of two different circles. They conclude the ratio of circumference to diameter (pi) remains constant in both situations.

Have students work with a partner in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- If the length of the radius is equal to 3 cm, what is the length of the diameter?
- If the length of the diameter is equal to 3 cm, what is the length of the radius?
- Would you expect the circumference of the circle with a 3 cm diameter to be half the circumference of the circle with a radius of 3 cm? Why or why not?
- Will circumference to diameter ratio be approximately 3.14 in any situation?

Differentiation strategy

To extend the activity, have students draw the two circles on graph paper. Then, have them investigate the relationship between the areas of the two circles. The areas of the circles can be estimated by counting square units.

Summary

The ratio of the circumference of a circle to its diameter is pi (π). You can use 3.14 or $\frac{22}{7}$ as approximations for π.