Lesson 5

Half Turns and Quarter Turns:

Rotations of Figures on the Coordinate Plane

Lesson Overview

Students use patty paper to explore rotations of various figures on a coordinate plane. They then generalize about the effects of rotating a figure on its coordinates. Students verify that two figures are congruent by describing a sequence of rigid motions that map one figure onto another

TEKS: 8.10A, 8.10C

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Jigsaw Transformations

Develop

Activity 5.1: Modeling Rotations on the Coordinate Plane

Day 2

Activity 5.2: Rotating Any Points on the Coordinate Plane
Activity 2.3: Verifying Congruence Using Rigid Motions

Demonstrate

Talk the Talk: Just the Coordinates

Getting Started: Jigsaw Transformations

Asynchronous Facilitation Notes

In this activity, students are presented with a jigsaw puzzle that has two pieces missing. Students match each missing piece to the open spot of the puzzle and describe the sequence of translations, reflections, and rotations that would move the puzzle piece presented to the matching open spot of the puzzle. **Note** that the transformations students list in this activity can be described informally.

Synchronous Facilitation Notes

In this activity, students are presented with a jigsaw puzzle that has two pieces missing. Students match each missing piece to the open spot of the puzzle and describe the sequence of translations, reflections, and rotations that would move the puzzle piece presented to the matching open spot of the puzzle.

Note that the transformations students list in this activity can be described informally—a reference to a line of reflection or center of rotation is not needed. You might want to discuss the difference between a horizontal flip and a vertical flip of a puzzle piece.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- Which rigid motion transformation will move the puzzle piece from its initial position to an open position on the puzzle board?
- Is there more than one rigid motion transformation or series of transformations that will work?
- Does the order of the transformations make a difference?
- Are the translations used to move the puzzle piece into position horizontal, vertical, or both?
- If a reflection is used to move the puzzle piece into position, where is the location of the line of reflection?
- If a rotation is used to move the puzzle piece into position, where is the location of the center of rotation?

Differentiation strategy

To scaffold support, suggest that students copy one or both of the puzzle pieces onto patty paper. They can then use the patty paper to perform the transformations necessary to fit the pieces into the puzzle.

Summary

Rigid motion transformations such as reflections, rotations, and translations can be used to describe and solve real-world situations.

Activity 5.1:
Modeling Rotations on the Coordinate Plane

Asynchronous Facilitation Notes

In this activity, students copy figures and the coordinates of their vertices onto patty paper. They perform rotations of 90° counterclockwise, 90° clockwise, 180 °, 270° clockwise, and 360° about the origin. Students record the coordinates of the original (pre-image) and rotated (image) figures. They explore how the rotations affect the coordinates of the pre-image to create the image. Students end the activity by making conjectures about the effects of rotations on an arbitrary ordered pair (x, y). If students do not have access to patty paper, they can use wax paper, parchment paper, tracing paper, or white paper. Students will need to upload images of their graphs.

Synchronous Facilitation Notes

In this activity, students copy figures and the coordinates of their vertices onto patty paper. They perform rotations of 90° counterclockwise, 90° clockwise, 180°, 270° clockwise, and 360° about the origin. Students record the coordinates of the original (pre-image) and rotated (image) figures. They explore how the rotations affect the coordinates of the pre-image to create the image. Students end the activity by making conjectures about the effects of rotations on an arbitrary ordered pair (x, y).

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

Questions to ask

- Why is a rotation of 180° considered a half turn?
- Is the figure rotated 180° clockwise about the origin or 180° counterclockwise about the origin? How do you know?
- What is the difference between rotating the figure 180° clockwise about the origin and rotating the figure 180° counterclockwise about the origin?
- Is there any relationship between the x-coordinate of the pre-image and the x-coordinate of the image? Is this true for all vertices?
- Is there any relationship between the y-coordinate of the pre-image and the y-coordinate of the image? Is this true for all vertices?
- Could this image have also been created by a translation? Explain.
- Could this image have also been created by a reflection? Explain.
- Why is a rotation of 90° considered a quarter turn?
- What is the difference between rotating the figure 90° clockwise about the origin and rotating the figure 90° counterclockwise about the origin?
- How are the coordinate changes evident in the position of the diagram?
- What quadrant(s) do you conjecture the image will lie in?
- How will the orientation of the image compare to the pre-image?
- What is the same about rotating a figure 90° counterclockwise about the origin and rotating a figure 270° about the origin?
- What do you notice about rotating a figure 360° about the origin?
- Can you think of any real-world examples of a 360° rotation?

Differentiation strategy

To extend the activity, have students (1) rotate the figure 90° clockwise about the origin, (2) reflect the figure across the x-axis, (3) reflect the figure across the y-axis, and (4) reflect the figure first across the x-axis and then across the y-axis. Compare these images with the two completed in the activity.

Summary

A point (x, y) rotated 180° about the origin becomes the point $(-x, -y)$, and when it is rotated 90° counterclockwise about the origin becomes the point $(-y, x)$. A point (x, y) rotated 270° clockwise about the origin becomes the point $(-y, x)$. A point rotated 360° about the origin becomes the point (x, y).

Activity 5.2:
Rotating Any Points on the Coordinate Plane

Asynchronous Facilitation Notes

A point with the coordinates (x, y) is located in the first quadrant. Students perform rotations of 90° counterclockwise, 90° clockwise, and 180° using the origin as the point of rotation and record the coordinates of the images in terms of x and y. Next, students begin with a triangle in Quadrant I and perform rotations of 90 degrees clockwise, 90 degrees counterclockwise, 180 degrees, 270 degrees clockwise, and 360 degrees. Lastly, they are given the coordinates of the vertices of a triangle, and without graphing they determine the coordinates of images resulting from different rotations. Students will need to upload images of their graphs. If students do not have access to patty paper, they can use wax paper, parchment paper, tracing paper, or white paper.

Synchronous Facilitation Notes

A point with the coordinates (x, y) is located in the first quadrant. Students perform rotations of 90° counterclockwise, 90° clockwise, and 180° using the origin as the point of rotation and record the coordinates of the images in terms of x and y. Next, students begin with a triangle in Quadrant I and perform rotations of 90 degrees clockwise, 90 degrees counterclockwise, 180 degrees, 270 degrees clockwise, and 360 degrees. Lastly, they are given the coordinates of the vertices of a triangle, and without graphing they determine the coordinates of images resulting from different rotations.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- When the figure was rotated 90° counterclockwise about the origin, did the rotation change the x-coordinate of each vertex?
- When the figure was rotated 90° counterclockwise about the origin, did the rotation change the y-coordinate of each vertex?
- Which coordinate in every point of the pre-image changed as a result of the 90° counterclockwise rotation about the origin?
- When the figure was rotated 90° clockwise about the origin, did the rotation change the x-coordinate of each vertex?

- When the figure was rotated 90° clockwise about the origin, did the rotation change the *y*-coordinate of each vertex?
- Does a 180° clockwise rotation about the origin bring about the same results as a 180° counterclockwise rotation about the origin?
- What rotations cause a switch between the *x*-coordinates and *y*-coordinates? Why do you think that is the case?
- If a rotation causes a switch in the *x*-coordinates and *y*-coordinates as well as a sign change, does it matter what order those changes are made? Explain.

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

Questions to ask

- If the point (x, y) is rotated 90° counterclockwise about the origin, how does the *x*-coordinate change?
- If the point (x, y) is rotated 90° counterclockwise about the origin, how does the *y*-coordinate change?
- If the point (x, y) is rotated 90° clockwise about the origin, how does the *x*-coordinate change?
- If the point (x, y) is rotated 90° clockwise about the origin, how does the *y*-coordinate change?
- If the point (x, y) is rotated 180° about the origin, how does the *x*-coordinate change?
- If the point (x, y) is rotated 180° about the origin, how does the *y*-coordinate change?
- If the rotation of a point (x, y) about the origin results in the point $(-x, y)$, what do you know about the rotation?
- If the rotation of a point (x, y) about the origin results in the point $(-y, x)$, what do you know about the rotation?
- If the rotation of a point (x, y) about the origin results in the point $(-x, -y)$, what do you know about the rotation?
- If the rotation of a point (x, y) about the origin results in the point $(y, -x)$, what do you know about the rotation?
- If the point (x, y) is rotated 270° about the origin, how does the *x*-coordinate change?
- If the point (x, y) is rotated 270° about the origin, how does the *y*-coordinate change?
- If the point (x, y) is rotated 360° about the origin, how does the *x*-coordinate change?
- If the point (x, y) is rotated 360° about the origin, how does the *y*-coordinate change?

Note that when students rotate the original triangle 270° about the origin, the coordinates are the same as the 90° counterclockwise rotation. When they rotate the original triangle 360° about the origin, the coordinates are the same as the original triangle.

Differentiation strategies

To scaffold support when students must consider rotations without graphing,

- Allow them to graph the vertices.
- Refer them to the general form at the start of the activity to make a connection to their work and the general form.

Summary

A point (x, y) when rotated 90° counterclockwise becomes the point $(-y, x)$, when rotated 90° clockwise becomes the point $(y, -x)$, and when rotated 180° becomes the point $(-x, -y)$.

Activity 5.3:
Verifying Congruence Using Rigid Motions

Asynchronous Facilitation Notes

In this activity, students examine the change in *x*- and *y*-coordinates to determine the congruence of geometric figures. They decide if a sequence of transformations can be used to prove the congruence of figures shown on a graph and then describe that sequence of rigid motions.

Synchronous Facilitation Notes

In this activity, students examine the change in *x*- and *y*-coordinates to determine the congruence of geometric figures. They decide if a sequence of transformations can be used to prove the congruence of figures shown on a graph and then describe that sequence of rigid motions.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategies

To scaffold support,

- Suggest that students use patty paper to determine the required transformations.
- Ask if the orientation of the figure remained the same, and if so, suggest they use translations (even if other rigid motions may work as well).

Questions to ask

- Is a translation involved in this situation? How do you know?
- Is a rotation involved in this situation? How do you know?
- Is a reflection involved in this situation? How do you know?
- What is another set of rigid motions that would create the same image?
- What is the *x*-coordinate of each point in the pre-image? What is the *x*-coordinate of each point in the image?
- Is the relationship between the *x*-coordinate of each point in the pre-image and its corresponding *x*-coordinate in the image the same for all pairs of corresponding points?
- What is the *y*-coordinate of each point in the pre-image? What is the *y*-coordinate of each point in the image?
- Is the relationship between the *y*-coordinate of each point in the pre-image and its corresponding *y*-coordinate in the image the same for all pairs of corresponding points?
- How can you check if you are correct?

Summary

Two figures are congruent if the same sequence of reflections, rotations, and translations moves all the points of one figure to all the points of the other figure.

Talk the Talk: Just the Coordinates

Asynchronous Facilitation Notes

In this activity, students use the coordinates of the pre-image and image to describe the rigid motion transformations associated with two congruent geometric figures. Encourage students to use graphs and patty paper and/or tables to visualize these problems.

Synchronous Facilitation Notes

In this activity, students use the coordinates of the pre-image and image to describe the rigid motion transformations associated with two congruent geometric figures.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategy

To assist all students, suggest they use graphs and patty paper and/or tables to visualize these problems.

Questions to ask

- What do you know to be true about the coordinates of points that have undergone a vertical translation?
- What do you know to be true about the coordinates of points that have undergone a horizontal translation?
- What do you know to be true about the coordinates of points that have undergone a reflection across the *x*-axis?
- What do you know to be true about the coordinates of points that have undergone a reflection across the *y*-axis?
- What do you know to be true about the coordinates of points that have undergone a 90° counterclockwise rotation about the origin?
- What do you know to be true about the coordinates of points that have undergone a 90° clockwise rotation about the origin?
- What do you know to be true about the coordinates of points that have undergone a 180° rotation about the origin?
- What is another set of rigid motion transformations to create this same image?

Summary

Rigid motion transformations can be used to verify the congruence of geometric figures.