Ratios
Topic 1 Overview

How is *Ratios* organized?

In this topic, students engage in high-level representational and definitional thinking about ratios. They focus on reasoning about ratios and laying a strong foundation for applying this reasoning in future topics and courses. Students begin the topic by associating ratios with multiplicative comparisons, contrasting them with additive comparisons. They learn about quantitative relationships represented by ratios and the different ways to represent ratios. Students are introduced to percent as a special ratio, a rate per 100. Students also explore qualitative relationships, those without numbers, to deepen their understanding of what it means to relate quantities. Exposure to qualitative comparisons removes the pressure for students to determine numerical answers, and instead encourages them to rely on their reasoning abilities.

Students use their initial understandings of ratio to model and determine equivalent ratios. To generate and display equivalent ratios in real-world and mathematical problems, they use tape diagrams, double number lines, scaling up and down, tables, and graphs. Students are expected to reason about why these strategies are valid. As students use different strategies, they consider the advantages and limitations of the strategies, but the focus remains on reasoning using number sense and the definition of ratio.

What is the entry point for students?

Students enter grade 6 with experience contrasting additive and multiplicative patterns and relationships. In prior grades, they wrote number sentences to represent multiplicative and additive scenarios. This topic draws on these past experiences in the opening lesson, requiring students to use and distinguish between additive and multiplicative reasoning to solve problems. From there, students are introduced to a specific type of multiplicative reasoning: ratio reasoning.

Students' knowledge of equivalent fractions from elementary school provides the foundation for their developing understanding of equivalent ratios. They have previously developed an understanding of fraction equivalence in terms of scaling, and they will use similar reasoning to create equivalent ratios by scaling up or scaling down a ratio using a scale factor.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Distinguish between additive and multiplicative relationships between quantities.
- Write ratios in words, with a colon, and in fractional form.
- Explain why order matters when writing a ratio.
- Write equivalent forms of ratios.
- Compare two qualitative and quantitative quantities with ratios.
- Determine when ratios represent part-to-whole relationships, part-to-part relationships, or rates.
- Use drawings, a double number line, and scaling up or down to generate equivalent ratios.
- Make a table of equivalent ratios using whole numbers.
- Determine the missing values in a table of equivalent ratios.
- Plot pairs of values that represent equivalent ratios on the coordinate plane.
- Recognize graphs of equivalent ratios as those forming a straight line that passes through the origin.
- Solve a variety of real-world and mathematical problems involving ratio and rate through reasoning with tables, tape diagrams, double number lines, graphs, and equations.

Why is *Ratios* important?

This topic provides the basis for future student learning of proportional relationships and slope. In grade 6, students build on these foundational ratio ideas in upcoming topics. As students work with expressions and equations in later module, they continue to distinguish between additive and multiplicative relationships and begin to use the more specific term *proportional relationships*. They use these distinctions as they compare and contrast equations and graphs of simple linear situations.

Although students are writing and solving problems by scaling proportions, students are not expected to solve ratio problems by cross multiplication, also called *means and extremes*. That strategy will be taught in grade 7. The strong emphasis on reasoning about ratios in grade 6

prepares students to understand an abstract algorithm.

In this topic, students graph equivalent ratios on the coordinate plane, a necessary preliminary for in-depth study of proportional relationships in grade 7. Reasoning about equivalent ratios prepares them to determine whether quantities are in a proportional relationship, identify a constant of proportionality, represent proportional relationships by equations, and explain the meanings of points on graphs of proportional relationships.

How do the activities in *Ratios* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

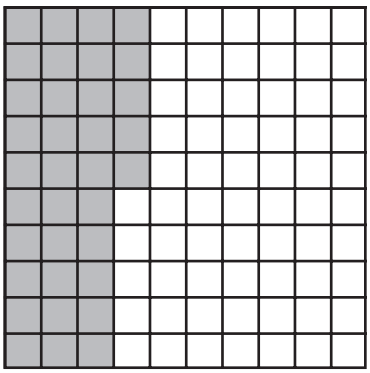
This topic highlights precision in language and mathematical notation, reasoning both qualitatively and quantitatively, and using a variety of tools and strategies to determine and display equivalent ratios. Students are expected to reason about ratio relationships while attending to precision in how they refer to the ratios in distinguishing between part-to-part and part-to-whole, using proper notation, and labeling the units on quantities. These habits of mind enable the class to communicate more effectively here and in future topics. Students learn a wide variety of strategies for determining equivalent ratios. As they develop their expertise around ratios, students are expected to weigh the advantages and limitations of the different strategies and choose the most efficient strategy for solving ratio problems.

Materials Needed

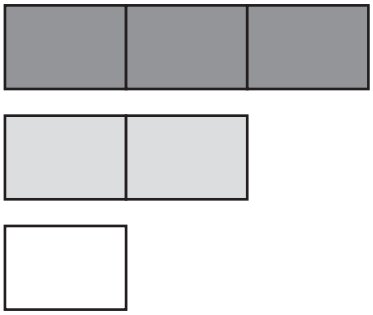
- Scissors
- Glue sticks or tape
- Poser paper

Concrete and Visual Representations Used

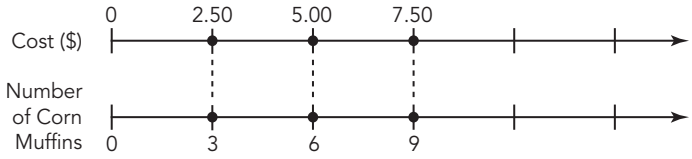
Hundredths Grid for Percent



Tape Diagrams to Determine Equivalent Ratios



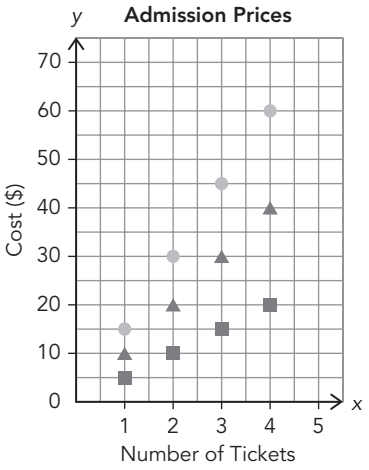
Double Number Lines to Determine Equivalent Ratios



Ratio Tables to Determine Equivalent Ratios

Weight on Earth (lbs)	60	30	90	120	150
Weight on the Moon (lbs)	10	5	15	20	25

Graphs of Equivalent Ratios



Learning Together

ELPS: 1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E, 5.F

Lesson	Lesson Name	TEKS	Days	Highlights
1	It's All Relative: Introduction to Ratio and Ratio Reasoning	6.4A 6.4C	3	Students differentiate between additive and multiplicative reasoning in preparation for the study of ratios. The term ratio is defined as a comparison between two quantities; ratios employ multiplicative reasoning. Students compare quantities using part-to-part and part-to-whole ratios. They write ratios in words, in colon notation, and in fractional form. They identify fractions and percents as special types of part-to-whole ratios.
2	Going Strong!: Comparing Ratios to Solve Problems	6.4B 6.4C	2	Students explore ratios in a different real-world situations. They decide which of two or more ratios in each situation is greater using qualitative and quantitative reasoning. Students compare part-to-part and part-to-whole ratios represented pictorially, verbally, and numerically. The focus in this lesson is on reasoning rather than on computation.
3	Oh, Yes, I Am the Muffin Man: Determining Equivalent Ratios	6.4B 6.4E 6.5A	3	Students are introduced to formal strategies to determine equivalent ratios, including pictures, tape diagrams, scaling up/down, and double number lines. They solve a variety of real-world problems using these strategies to create equivalent equations. An example of scaling up ratios is provided, and students use the example to answers questions in various contexts. The definitions of scaling up and scaling down ratios are provided. Students then determine equivalent ratios by either scaling up or scaling down. A double number line is introduced. They use double number lines to represent the proportional relationship between two quantities and solve for unknown quantities.
4	A Trip to the Moon: Using Tables to Represent Equivalent Ratios	6.4B 6.4D 6.5A	2	Students use tables in different ways to determine equivalent ratios. They multiply or divide existing ratios by a common factor to determine equivalent ratios in a table, just as they did in scaling. Students learn that existing ratios in a ratio table can be added to form new equivalent ratios. They then complete equivalent ratio tables for different proportional situations.

Lesson	Lesson Name	TEKS	Days	Highlights
5	They're Growing!: Graphs of Ratios	6.4E 6.5A 6.6C	2	Students investigate rectangles with a common ratio of side lengths and those with a constant difference in side lengths. They graph the dimensions of the rectangles on a coordinate plane and conclude that equivalent ratios represented on the coordinate plane form a straight line that passes through the origin. Students analyze a ratio that is represented using a table, double number line, and coordinate plane. The models are connected and used to solve real-world problems.
6	One Is Not Enough: Using and Comparing Ratio Representations	6.5A 6.6C	3	Graphs and double number lines of real-world situations are given. Students interpret the points on the graphs in terms of the problem situation. They determine unknown ratios using either a specified strategy or the strategy of their choice. Students also contrast representations of additive and multiplicative relationships. Students then create a graphic organizer to show how equivalent ratios can be modeled through four representations: scale up/scale down, tables, double number lines, and graphs.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 6.4A, 6.4C **LESSON 1** **It's All Relative** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 1** continued ACTIVITY 3 ACTIVITY 4	**LESSON 1** continued ACTIVITY 5 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 6.4B, 6.4C **LESSON 2** **Going Strong** GETTING STARTED ACTIVITY 1 ACTIVITY 2
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LESSON 2 continued ACTIVITY 3 TALK THE TALK	TEKS: 6.4B, 6.4E, 6.5A **LESSON 3** **Oh, Yes, I Am the Muffin Man** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 3** continued ACTIVITY 3 ACTIVITY 4	**LESSON 3** continued ACTIVITY 5 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
MID-TOPIC ASSESSMENT	TEKS: 6.4B, 6.4D, 6.5A **LESSON 4** **A Trip to the Moon** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 4** continued ACTIVITY 3 TALK THE TALK	TEKS: 6.4E, 6.5A, 6.6C **LESSON 5** **They're Growing** GETTING STARTED ACTIVITY 1	**LESSON 5** continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK
Day 16	**Day 17**	**Day 18**	**Day 19**	**Day 20**
MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 6.5A, 6.6C **LESSON 6** **One Is Not Enough** GETTING STARTED ACTIVITY 1	**LESSON 6** continued ACTIVITY 2 ACTIVITY 3	**LESSON 6** continued ACTIVITY 4 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress
Day 21				
END OF TOPIC ASSESSMENT				

Assessments

There are two assessments aligned to this topic: Mid-Topic Assessment and End of Topic Assessment.