TEXAS MATH SOLUTION

Grade 6

Student Edition

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with Kelly Edenfield, Josh Fisher,
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"Mathematics is so much more than memorizing rules. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing(TM)—you need to actively engage with the content if you are to benefit from it. The lessons were designed to take you from your intuitive understanding of the world and build on your prior experiences to then learn new concepts. My hope is that these instructional materials help you build a deep understanding of math."

Sandy Bartle Finocchi, Chief Mathematics Officer

"My hope is that as you work through this course, you feel capable—capable of exploring new ideas that build upon what you already know, capable of struggling through challenging problems, capable of thinking creatively about how to fix mistakes, and capable of thinking like a mathematician."

Amy Jones Lewis, Senior Director of Instructional Design

"At Carnegie Learning we have created an organization whose mission and culture is defined by your success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in you. Our hope is that you will enjoy our resources as much as we enjoyed creating them."

Barry Malkin, CEO

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Glossary

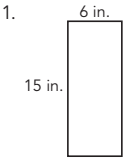
1 Taking Apart Numbers and Shapes

Writing Equivalent Expressions Using the Distributive Property

REVIEW

Calculate the area of each rectangle. Show your work.

1.



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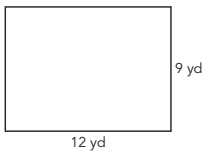


LEARNING GOALS 1

- Write, read, and evaluate equivalent numeric expressions.
- Identify the adjacent side lengths of a rectangle as factors of the area value.
- Identify parts of an expression, such as the product and the factors.
- Write equivalent numeric expressions for the area of a rectangle by decomposing one side length into the sum of two or more numbers.
- Apply the Distributive Property to rewrite the product of two factors.

KEY TERMS

- numeric expression
- equation
- Distributive Property

2 You know how to operate with numbers using different strategies. Taking apart numbers before you operate can highlight important information or make calculations easier. How can you use these strategies to express number sentences in different ways?

1. Learning Goals

Learning goals are stated for each lesson to help you take ownership of the learning objectives.

2. Connection

Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

3. Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

3 Getting Started

Break It Down to Build It Up

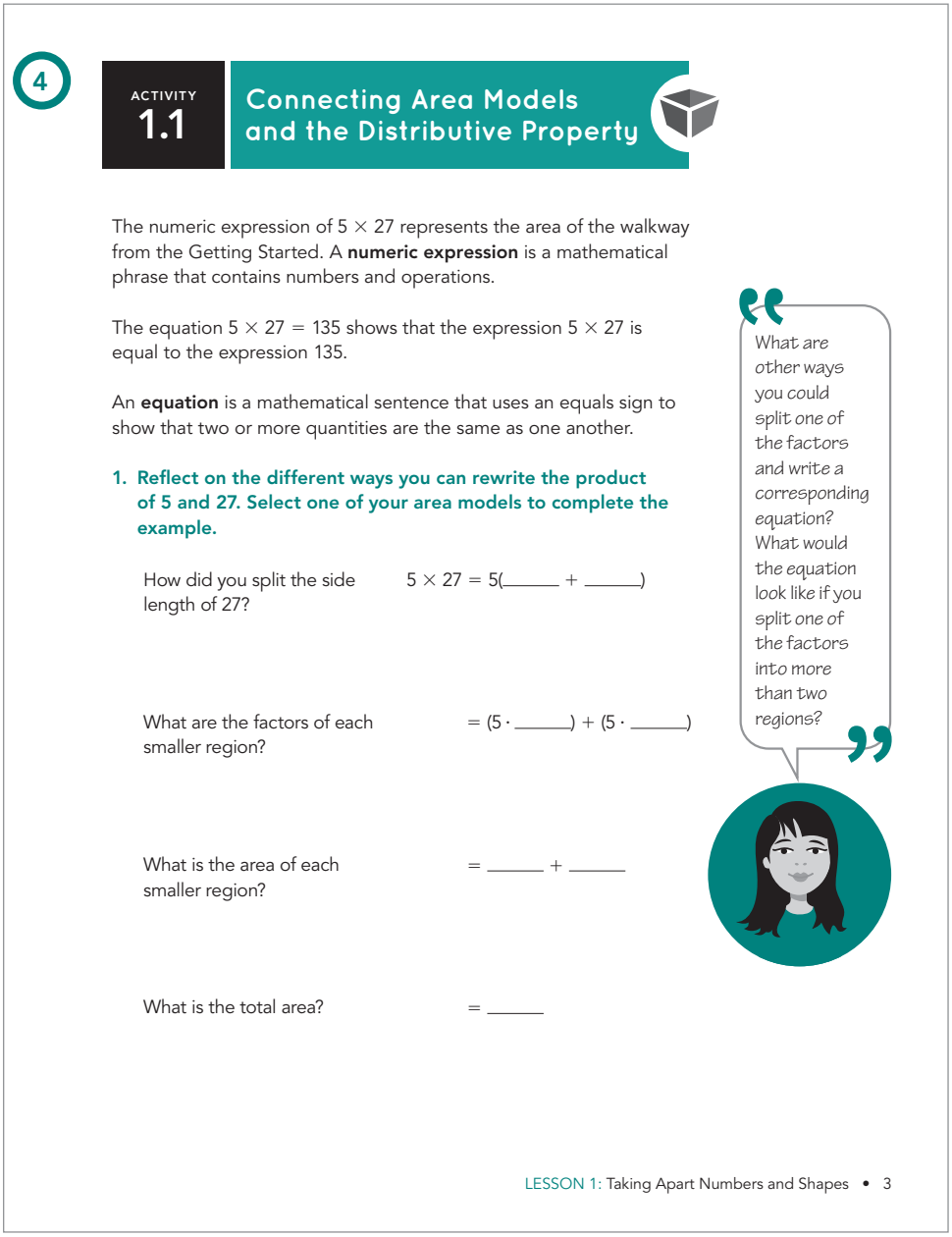
Callie is building a rectangular walkway up to her house. The width of the walkway is 5 feet and the length is 27 feet. She needs to calculate the area of the walkway to determine the amount of materials needed to build it.

1. **Mark and label 2 different ways you could divide an area model to determine the area of the walkway.**

2. **Determine the areas of each of the subdivided parts of your models.**

3. **What is the total area of the walkway?**

2 • TOPIC 1: Factors and Multiples



4

ACTIVITY 1.1

Connecting Area Models and the Distributive Property

The numeric expression of 5×27 represents the area of the walkway from the Getting Started. A **numeric expression** is a mathematical phrase that contains numbers and operations.

The equation $5 \times 27 = 135$ shows that the expression 5×27 is equal to the expression 135.

An **equation** is a mathematical sentence that uses an equals sign to show that two or more quantities are the same as one another.

1. Reflect on the different ways you can rewrite the product of 5 and 27. Select one of your area models to complete the example.

How did you split the side length of 27?

$5 \times 27 = 5(\_\_\_\_\_ + \_\_\_\_\_)$

What are the factors of each smaller region?

$= (5 \cdot \_\_\_\_\_) + (5 \cdot \_\_\_\_\_)$

What is the area of each smaller region?

$= \_\_\_\_\_ + \_\_\_\_\_$

What is the total area?

$= \_\_\_\_\_$

What are other ways you could split one of the factors and write a corresponding equation? What would the equation look like if you split one of the factors into more than two regions?

LESSON 1: Taking Apart Numbers and Shapes • 3

4. Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

5. Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

NOTES

5

TALK the TALK

The Floor Is Yours

You can apply the Distributive Property to solve real-world problems.

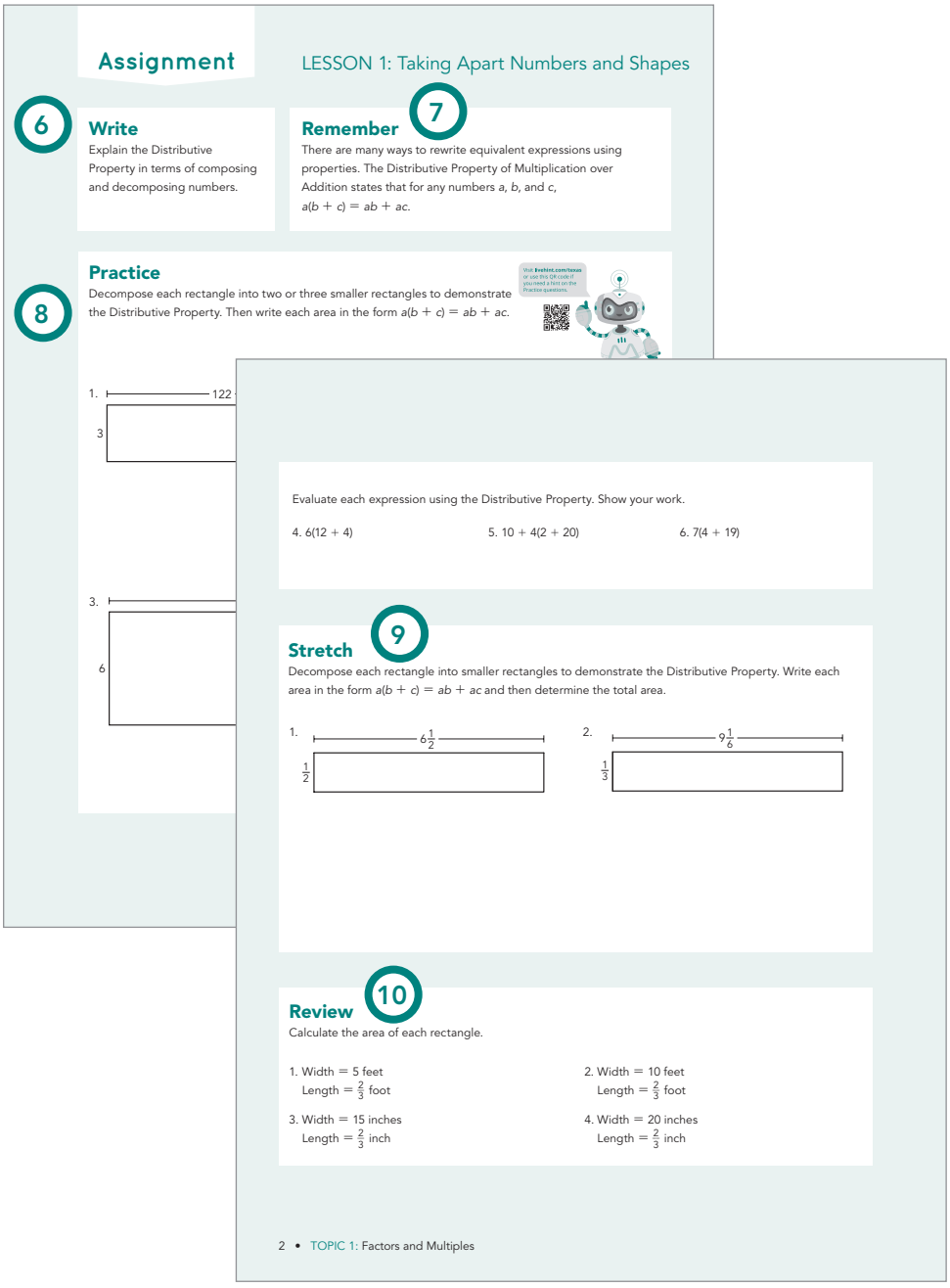
Consider the situation.

Tyler is setting up the gym floor for an after-school program. He wants to include a rectangular area for playing volleyball and another for dodgeball. He also wants to have an area for kids who like to play board games or just sit and read. The gym floor is already 50 feet by 84 feet, or 4200 square feet.

1. **Create a diagram to show how you would split up the gym floor. Represent your diagram using the Distributive Property and write an explanation for the areas assigned to each activity.**

6 • TOPIC 1: Factors and Multiples

Assignment



Assignment

LESSON 1: Taking Apart Numbers and Shapes

6 **Write**

Explain the Distributive Property in terms of composing and decomposing numbers.

7 **Remember**

There are many ways to rewrite equivalent expressions using properties. The Distributive Property of Multiplication over Addition states that for any numbers *a*, *b*, and *c*, $a(b + c) = ab + ac$.

8 **Practice**

Decompose each rectangle into two or three smaller rectangles to demonstrate the Distributive Property. Then write each area in the form $a(b + c) = ab + ac$.

1. 122, 3

3. 6

Evaluate each expression using the Distributive Property. Show your work.

4. 6(12 + 4)

5. 10 + 4(2 + 20)

6. 7(4 + 19)

9 **Stretch**

Decompose each rectangle into smaller rectangles to demonstrate the Distributive Property. Write each area in the form $a(b + c) = ab + ac$ and then determine the total area.

1. $6\frac{1}{2}$, $\frac{1}{2}$

2. $9\frac{1}{6}$, $\frac{1}{3}$

10 **Review**

Calculate the area of each rectangle.

1. Width = 5 feet
   Length = $\frac{2}{3}$ foot

2. Width = 10 feet
   Length = $\frac{2}{3}$ foot

3. Width = 15 inches
   Length = $\frac{2}{3}$ inch

4. Width = 20 inches
   Length = $\frac{2}{3}$ inch

2 • TOPIC 1: Factors and Multiples

6. Write

Reflect on your work and clarify your thinking.

7. Remember

Take note of the key concepts from the lesson.

8. Practice

Use the concepts learned in the lesson to solve problems.

9. Stretch

Ready for a challenge?

10. Review

Remember what you've learned by practicing concepts from previous lessons and topics.

Problem Types You Will See

Worked Example

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

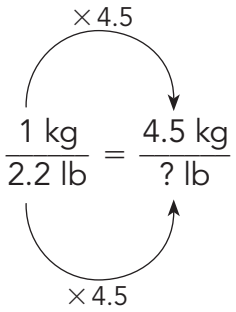
Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

WORKED EXAMPLE

Determine the quantity in pounds that is equivalent to 4.5 kilograms.

Scaling Up



Unit Analysis

$$4.5 \text{ kg} \left(\frac{2.2 \text{ lb}}{1 \text{ kg}}\right)$$

$$\frac{4.5 \cancel{\text{kg}}}{1} \left(\frac{2.2 \text{ lb}}{1 \cancel{\text{kg}}}\right) = 9.9 \text{ lb}$$

$$\frac{1 \text{ kg}}{2.2 \text{ lb}} = \frac{4.5 \text{ kg}}{9.9 \text{ lb}}$$

$$4.5 \text{ kg} = 9.9 \text{ lb}$$

Christopher and Max want to determine the number of miles in 31,680 feet using unit analysis.

Max

$$31{,}680 \cancel{\text{ft}} \left(\frac{1 \text{ mi}}{5280 \cancel{\text{ft}}}\right) = 6 \text{ mi}$$

Christopher

$$31{,}680 \text{ ft} \left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) = 167{,}270{,}400 \text{ mi}$$

Thumbs Up

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

Ask Yourself:

- Why is this method correct?
- Have I used this method before?

Thumbs Down

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

Tim and Dan love cereal, but don't want spend a lot of money. After scanning the aisle in the grocery store for the lowest prices, the boys make the following statements.

- **Tim says, "I found Sweetie Oat Puffs for $0.14 per ounce. That's the cheapest cereal in the aisle!"**
- **Dan replies, "It's not cheaper than Sugar Hoops! The unit price for that is 6.25 oz per dollar."**

Who is correct? Explain your reasoning.

Who's Correct

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine correct or not correct.

Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

The Crew

The Crew is here to help you on your journey. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.

Mathematical Process Standards

Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I can" expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

▶ Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

▶ Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem solving process and reasonableness of the solution.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

▶ Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

▶ Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

▶ Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

▶ Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure
- look for general methods and more efficient ways to solve problems.

▶ Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

Academic Glossary

Visit the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

ANALYZE

Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

EXPLAIN YOUR REASONING

Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

REPRESENT

Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

Related Phrases

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

ESTIMATE

Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

Related Phrases

- Predict
- Approximate
- Expect
- About how much?

DESCRIBE

Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?