## Lesson 4

# Putting the V in Absolute Value:

### Defining Absolute Value Functions and Transformations

### Lesson Overview

Students are already familiar with the general shape of the graphs of absolute value functions, and they have studied transformations of linear functions. In this lesson, students experiment with the absolute value function family. They expand their understanding of transformations to include horizontal translations and dilations. Students interpret functions in the form $f(x) = A(B(x - C) + D$. They distinguish between the effects of changing values inside the argument of the function (the *B*- and *C*-values) and changing values outside the function (the *A*- and *D*-values). At the end of the lesson, students summarize the impact of transformations on the domain and range of the absolute value function.

**Lesson Video(s):** The aligned lesson overview video(s) provide additional instruction for students on the key concepts in this lesson and can be found alongside the digital interactive student lesson.

**TEKS:** 2A.2A, 2A.6C, 2A.7I

## Lesson Structure and Pacing: 3 Days

### Day 1

**Engage**

Getting Started: Distance Is Always Positive

**Develop**

Activity 4.1: Graphs of Absolute Value Functions

### Day 2

Activity 4.2: Transformations Outside the Function
Activity 4.3: Transformations Inside the Function

### Day 3

Activity 4.4: Combining Transformations of Absolute Value Functions
Activity 4.5: Writing Equations in Transformation Form

**Demonstrate**

Talk the Talk: *A*, *B*, *C*, and *D*

# Getting Started: Distance Is Always Positive

## Asynchronous Facilitation Notes

In this activity, *absolute value* is defined, and students recall how to evaluate absolute value expressions prior to exploring absolute value functions in the next activity. The students will explore eight different absolute value expressions. They will model each absolute value expression on the *x*-axis of the coordinate plane using the free draw tool and then rewrite each expression without the absolute value symbol in a fill in the blank question. Students also address a common misconception that the expression $-x$ always represents a negative number in a free response question.

## Synchronous Facilitation Notes

In this activity, *absolute value* is defined, and students recall how to evaluate absolute value expressions prior to exploring absolute value functions in the next activity. They model different absolute value expressions on the *x*-axis of a coordinate plane. Students also address a common misconception that the expression $-x$ always represents a negative number.

Complete Questions 1 through 3 as a class. Be sure that students use the coordinate plane to model the expression, not just the final result. For example, for $|-2|$, students should start at $-2$ and then reflect across 0 to get a result of $+2$.

**Differentiation strategies**

- To assist all students, have students circle 0 on the *x*-axis of the coordinate plane so that the distance from zero is more explicit.
- To extend the activity, have students represent these equations on the number line: $|x| = 3$, $|x| = 5$, $|x| = -2$, $|x - 2| = 5$, and $|x + 2| = 4$.

**Misconceptions**

- Students may incorrectly assume that $-a$ always represents a negative value. To clarify this error in thinking, remind students to read the expression *-x as the opposite of x*.
- Students may have memorized that the answer is always positive in absolute value problems, but it is important that they understand that absolute value is a measure of distance from zero as they deal with the absolute value function in later activities.

**As students work, look for**

Sign errors when simplifying the expressions.

**Questions to ask**

- Why did every expression go to the positive numbers on the number line?
- Why is the absolute value of an expression always positive or zero?
- How should the equation $|a| = -a$ be read?
- Explain what $|a| = -a$ means in your own words.
- Do you always take the opposite of a number when taking its absolute value? Explain.

## Summary

The absolute value of any numeric expression is a positive number or zero, representing the number of units that value is from zero on the number line.

# Activity 4.1:
## Graphs of Absolute Value Functions

### Asynchronous Facilitation Notes

In this activity, students model the functions $f(x) = x$, $f(x) = |x|$, $f(x) = -x$ and $f(x) = |-x|$ on a coordinate plane using the free draw feature. They also record the values for each function in a fill in the blank table, graph them on a coordinate plane, and compare the functions and graphs in free response questions. Remind students that they can determine ordered pairs in a table using technology or algebraically using the function. Through this activity, students make sense of the basic shape of an absolute value function and its limited range.

### Synchronous Facilitation Notes

In this activity, students model of the functions $f(x) = x$, $f(x) = |x|$, $f(x) = -x$ and $f(x) = |-x|$ on a coordinate plane. They also record the values for each function in a table, graph them on a coordinate plane, and compare the functions and graphs. Through this activity, students make sense of the basic shape of an absolute value function and its limited range.

**Differentiation strategy**

Have students sketch the graphs of $f(x) = |x|$ and $f(x) = -|x|$ in a different color than than the functions of $f(x) = x$ and $f(x) = -x$. This way they will be able to tell the two functions apart on the coordinate plane. The use of different colors will be helpful throughout this lesson as students place multiple graphs on the same coordinate plane.

**As students work, look for**

A discrete graph rather than a continuous graph. Some graphs may contain only the values in the table rather than all points on the entire function. Emphasize the domain of the function includes all real numbers.

**Questions to ask**

- What are the domain and range of $f(x) = x$?
- Which values needed to move on the coordinate plane when graphing the absolute value function? Why did they have to move while others did not?
- Which quadrants of the coordinate plane contain positive $x$-values? Positive $y$-values?
- Which quadrants contain the absolute value function? Why is that the case?
- What are the domain and range for $f(x) = |x|$?

Complete Questions 5 through 8 as a class.

**Questions to ask**

- Which values needed to move on the coordinate plane when graphing $f(x) = |-x|$? Why did they have to move while others did not?

- Given a negative *x*-value, how did you determine the location for the graph of $f(x) = |-x|$?
- Given a positive *x*-value, how did you determine the location for the graph of $f(x) = |-x|$?
- What are the domain and range for $f(x) = |-x|$?
- Is an absolute value function also a linear function? Why or why not?
- Are absolute value functions considered increasing functions or decreasing functions? Explain.

### Summary

The basic absolute value function is a V-shaped graph. Except for (0, 0), all points on the graph have positive *y*-coordinates.

## Activity 4.2:
**Transformations Outside the Function**

### Asynchronous Facilitation Notes

In this activity, students connect what they know about linear function transformations to translate and dilate absolute value functions. For each transformation type, students write the new function in terms of the basic function in free response questions, graph the function using technology and the free draw feature, and then generalize the effect of the *A*- or *D*-value in the new equation both verbally in free response questions and by using coordinate notation to represent the transformation in fill in the blank questions. Students then analyze student work and list the step by step reasoning each student used when sketching multiple-step transformations of an absolute value function, filling in the blanks for each step. Students explain how the order of transformations affects the line of reflection in a free response question. Students are then given absolute value functions in transformation form, they use coordinate notation to describe the transformation in free response questions and graph the transformations using the free draw feature. They also describe transformations in terms of other transformations in free response questions and explain how the *A*- and *D*- values affect the minimum and maximum values of absolute value functions in a free response question.

### Synchronous Facilitation Notes

In this activity, students connect what they know about linear function transformations to translate and dilate absolute value functions. Students explain how the order of transformations affects the line of reflection. For each transformation type, students write the new function in terms of the basic function, graph the function using technology, and then generalize the effect of the *A*- or *D*-value in the new equation.

**Differentiation strategy**

- As an alternative grouping method, use the jigsaw strategy for this activity. The table shown demonstrates how to organize a class of 30 students, with 3 student names per cell.

| | Group A Questions 1–4 | Group B Questions 5–7 |
|---|---|---|
| Group 1 | | |
| Group 2 | | |
| Group 3 | | |
| Group 4 | | |
| Group 5 | | |

- Have each group (column) complete their assigned questions. Then, regroup by group numbers (rows), so that there are groups of 6 students with pairs from Groups A and B. Give each pair a time limit to be the teacher to their peers for their set of questions. Then, have all groups read the information prior to Question 8, complete Questions 8 through 15, and discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

**Differentiation strategy**

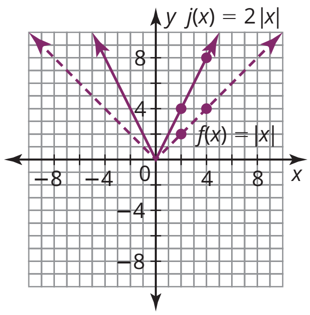
To scaffold instruction and assist students with the process of transferring the graph from their technology to their own coordinate plane, suggest they access the table of values on their technology for help.

**Questions to ask**

- What is the shape of the absolute value function?
- Where is the absolute value function on your calculator?
- What is a vertical translation?
- How does a vertical translation affect each point on the graph of the original function?
- Does a vertical translation change the shape of a function?
- How do you know whether the vertical translation is a shift up or down?
- How are vertical translations apparent in the algebraic representation of a function?
- Write a function that will translate the absolute value function up 4 units.

- How are vertical translations apparent in the graphic representation of a function?
- Considering a vertical translation of $D$, does each point $(x, y)$ on the graph of $g(x)$ become $(x, y - D)$ or $(x, y + D)$?

Have students work with a partner or in a group to complete Questions 5 through 7. Share responses as a class.




**Misconception**
Students may observe the graph of $j(x) = 2|x|$ and identify the change in the graph as a horizontal compression rather than a vertical stretch. Clarify this misconception now using the example provided. For example, given $j(x) = 2|x|$. The $y$-value is multiplied by 2. Point (2, 2) moves to point (2, 4), and point (4, 4) moves to point (4, 8). It is not the case that point (4, 4) moves to the point (2, 4).

**Questions to ask**
- What is a vertical dilation?
- How does a vertical dilation affect each point on the graph of the original function?
- Does a vertical dilation change the shape of a function?
- How do you know whether the vertical dilation is a vertical stretch or a vertical compression?
- What dilations cause a vertical stretching of the function?
- What dilations cause a vertical compression of the function?
- How are vertical dilations apparent in the algebraic representation of a function?
- How are vertical dilations apparent in the graphic representation of a function?
- Considering a vertical dilation of factor $A$, does each point $(x, y)$ on the graph $g(x)$ become $(x, Ay)$ or $(Ax, y)$?

Have students work with a partner or in a group to complete Questions 8 and 9. Share responses as a class.

**Questions to ask**
- What is a line of reflection? A line of symmetry?
- What is the difference between a line of reflection and a line of symmetry?
- Are lines of reflection always the $x$- or $y$-axis?
- How are Josh's steps and Vicki's steps different from one another?
- What line of reflection did Josh use? What line of reflection did Vicki use?
- Did Josh and Vicki end up with the same result?
- Does a reflection or a translation change the shape of the function?
- Does a reflection followed by a shift result in the same graph as the same shift followed by a reflection?
- Do you think the order of transformations ever makes a difference in the graph of a function? If so, for what transformations?

Have students work with a partner or in a group to complete Questions 10 through 15. Share responses as a class.

**As students work, look for**

Errors calculating $b(x)$ by applying the transformations to $a(x)$ rather than $f(x)$.

**Misconceptions**

- Students may think that all reflections are the same, not realizing the line of reflection makes a difference. Clarify this misconception by comparing $a(x)$ and $-a(x)$, where the $x$-axis is the line of reflection, and $a(x)$ and $b(x)$, where $y = 1$ is the line of reflection. Use patty paper with the axes drawn on it to demonstrate how $b(x)$ and $-a(x)$ are created from reflections of $a(x)$.
- When describing the line of reflection as the $x$-axis, students may identify it as $x = 0$ rather than $y = 0$.

**Questions to ask**

- Which transformations must be performed on $f(x)$ to create $a(x)$?
- What is the point (2, 2) from $f(x)$ mapped onto in $a(x)$?
- Would the graph of $a(x)$ look different if the vertical translation occurred before the vertical dilation? If so, how?
- What is the point (2, 2) from $f(x)$ mapped onto in $b(x)$?
- Is the graph of $a(x)$ reflected across the $x$-axis to create $b(x)$?
- Is the graph of $a(x)$ reflected across the line $y = 1$ to create $b(x)$?
- What is the line of reflection to create $-a(x)$ from $a(x)$

## Summary

Absolute value functions can be transformed similar to linear functions. Changing the $D$-value vertically translates the graph of the function. Changing the $A$-value vertically dilates and/or reflects the graph of the function.

# Activity 4.3:

## Transformations Inside the Function

## Asynchronous Facilitation Notes

Students investigate horizontal translations, which occur by changing the argument inside absolute value functions. They then connect what they know about vertical dilations to horizontal dilations. For each transformation type, students write the new function in terms of the basic function in free response questions, graph the function using technology and the free draw feature, and then generalize the effect of the $C$- or $B$-value in the new equation verbally in free response questions and by using coordinate notation to represent the transformation in fill in the blank questions.

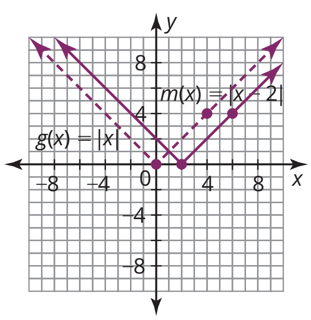
## Synchronous Facilitation Notes

Students investigate horizontal translations, which occur by changing the argument inside absolute value functions. They then connect what they know about vertical dilations to horizontal dilations. For each transformation type, students write the new function in terms of the basic function, graph the function using technology, and then generalize the effect of the *C*- or *B*-value in the new equation.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

**Misconception**

Students may overgeneralize what they know about vertical translations to horizontal translations. They may think that the graph of $m(x) = |x - 2|$ is a horizontal shift from 0 to −2 and that the graph of $m(x) = |x + 2|$ is a horizontal shift of the function from 0 to +2. Clarify this misconception by helping students reason through plotting points as explained in the example shown. Follow-up by discussing why the *C*-value is being subtracted in the function $t(x) = f(x - C)$.




Given $m(x) = |x - 2|$. The expression $x - 2$ means that 2 is subtracted from each *x*-value before the absolute value function is applied.

Therefore, every value of *x* must be increased by 2 to get back to the original function.

So, if $x = 2$, the expression inside the absolute value symbol is only 0.

The point (2, 0) in the transformed function corresponds to point (0, 0) in the original function.

The point (0, 0) moves onto point (2, 0). The point (4, 4) moves onto point (6, 4).

**Questions to ask**

- What is a horizontal translation?
- How does a horizontal translation affect each point on the graph of the original function?
- Does a horizontal translation change the shape of a function?
- What translation causes a shift to the left? To the right?
- How are horizontal translations apparent in the algebraic representation of a function?
- How are horizontal translations apparent in the graphic representation of a function?
- Considering a horizontal translation of *C*, does each point $(x, y)$ on the graph become $(x + C, y)$ or $(x - C, y)$?

**Differentiation strategy**

To extend the activity, have students create an informational classroom slide for each transformation of an absolute value function.

Have students work with a partner or in a group to complete Questions 4 through 6. Share responses as a class.

**Misconception**

Students may observe the graph of $j(x) = 2|x|$ and identify the change in the graph as a vertical stretch rather than a horizontal compression. Clarify this misconception now using the example provided. For example, given $j(x) = 2|x|$. The $x$-value is multiplied by $\frac{1}{2}$. Point (2, 2) moves to point (1, 2), and point (4, 4) moves to point (2, 4). It is not the case that point (4, 4) moves to the point (4, 8).

**Questions to ask**

- What is a horizontal dilation?
- How does a horizontal dilation affect each point on the graph of the original function?
- Does a horizontal dilation change the shape of a function?
- How do you know whether the horizontal dilation is a horizontal stretch or a horizontal compression?
- What dilations cause a horizontal stretching of the function?
- What dilations cause a horizontal compression of the function?
- How are horizontal dilations apparent in the algebraic representation of a function?
- How are horizontal dilations apparent in the graphic representation of a function?
- Considering a horizontal dilation of factor $B$, does each point $(x, y)$ on the graph $g(x)$ become $(x, \frac{1}{B}y)$ or $(\frac{1}{B}x, y)$?

## Summary

Like linear and quadratic functions, absolute value functions can be transformed. Changing the $C$-value horizontally translates the graph of the function. Changing the $B$-value horizontally dilates and/or reflects the graph of the function.

# Activity 4.4:
## Combining Transformations of Absolute Value Functions

### Asynchronous Facilitation Notes

In this activity, students distinguish a horizontal translation or dilation, which occurs by changing the $B$- and $C$-value inside the function, from vertical translations and dilations, which occur by changing values outside the function. Given functions of the form $g(x) = A \cdot f(B(x - C)) + D$, students use coordinate notation to describe all three transformations in free response questions and then graph each function using the free draw feature.

### Synchronous Facilitation Notes

In this activity, students distinguish a horizontal translation or dilation, which occurs by changing the $B$- or $C$-value inside the function, from vertical translations and dilations, which occur by changing values outside the function. Given functions of the form $g(x) = A \cdot f(B(x - C)) + D$, students use coordinate notation to describe all three transformations and then graph each function.

Ask a student to read the introduction aloud. Discuss as a class. Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Ask a student to read the paragraph before Question 2 and discuss as a class.

**Questions to ask**

- How does the ordered pair $(x, |x|)$ describe any point on the basic absolute value function?
- How does the coordinate notation of the transformed absolute value function relate to the coordinate notation you wrote in Question 1?

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

**As students work, look for**

Sign errors associated with transformations that affect the *x*-coordinate (input) of a function and the *y*-coordinate (output) of the function.

**Questions to ask**

- How does an *A*-value equal to $-1$ affect the graph of the function?
- How does a *C*-value equal to $-1$ affect the graph of the function?
- How does a *D*-value equal to $-1$ affect the graph of the function?
- Which function(s) include a vertical dilation? How can you tell?
- Which function(s) include a vertical translation? How can you tell?
- Which function(s) include a horizontal translation? How can you tell?
- Which function(s) include a reflection? How can you tell?
- What is a general equation for a function that has no vertical translation?
- What is a general equation for a function that has no horizontal translation?
- What is a general equation for a function that has no vertical dilation?

**Differentiation strategy**

To extend the activity, play *Guess my Function* with the class. Begin by defining a function as $f(x) = |x|$, then write the general equation $g(x) = A \cdot f(B(x - C)) + D$ on the board. Tell students you are thinking of a function that has a vertical translation of $-4$. Ask them to write this function as an equation. Then tell them this function also is vertically stretched by a factor of 7, and ask that they rewrite their equation to include this transformation. Then tell them the function was also reflected across the *x*-axis. Ask them to compare their final equations with their classmates' equations. It should be $g(x) = -7 \cdot f(x) - 4$. Play a few rounds of this game for additional practice.

## Summary

Given a function of the form $f(x) = A \cdot f(B(x - C)) + D$, a horizontal translation occurs by changing the *C*-value inside the function argument, while vertical dilations and translations occur by changing the values of *A* and *D*, respectively, outside the function. Changing the *B*-value results in a horizontal dilation.

# Activity 4.5:
## Writing Equations in Transformation Form

### Asynchronous Facilitation Notes

In this activity, students practice writing functions in transformation function form in terms of their specific transformations, then write an equivalent equation for each. The students will do this activity by filling in blanks in a provided table. Make sure they understand the difference between transformation function form and its equivalent equation.

### Synchronous Facilitation Notes

In this activity, students practice writing functions in transformation function form in terms of their specific transformations, then write an equivalent equation for each.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

**Questions to ask**

- Is a reflection across the $x$-axis shown in the argument of the function or outside of the function? Why?
- Is a horizontal translation shown in the argument of the function or outside of the function? Why?
- How can you tell whether a vertical dilation is a compression or stretch?
- Why is a vertical compression expressed as a proper fraction?

### Summary

Vertical translations, horizontal translations, and vertical dilations of functions can be combined to transform absolute value functions.

# Talk the Talk: A, C, and D

### Asynchronous Facilitation Notes

In this activity, students interpret translations of the function $f(x) = A \cdot f(B(x - C)) + D$ by making sense of the values $A$, $B$, $C$, and $D$ expressed in general terms. First, they relate the values of the parameters to their graphs in a matching activity. Next, they will fill in blanks to complete a table to describe the various transformations of absolute value functions. Students also address true/false statements about the effects of transformations on domain and range in free response questions. For each statement they identify as false, they will rewrite the statement as true.

### Synchronous Facilitation Notes

In this activity, students interpret translations of the function $f(x) = A \cdot f(B(x - C)) + D$ by making sense of the values $A$, $B$, $C$, and $D$ expressed in general terms. First, they relate the values of the parameters to their graph. Students also address true/false statements about the effects of transformations on domain and range.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

**As students work, look for**

- Difficulty dealing with the values of *A*, *B*, *C*, and *D* expressed in general terms. If that is the case, suggest students rephrase statements in their own words, such as stating "*D* is positive" or "*A* is a proper fraction."
- Sign errors related to horizontal translations.

**Questions to ask**

- How does knowing whether *A* is less than 0, between 0 and 1, or greater than 1 help you describe the vertical dilation of a function?
- How does knowing whether *C* is positive or negative help you describe the horizontal translation of a function?
- How does knowing whether *D* is positive or negative help you describe the vertical translation of a function?
- Which values are associated with a vertical translation? Horizontal translation? A stretch? A compression?

**Differentiation strategy**

To extend the activity, ask students to create a graphical representation to support each of the true statements or rewritten false statements in Question 3.

## Summary

Knowing the sign of the values of *A*, *B*, *C*, and *D* in $y = A \cdot f(B(x - C)) + D$ helps to inform the type of transformation to be performed on the function.