Module 1 Overview

Exploring Patterns in Linear and Quadratic Relationships

"Linear programming can be viewed as part of a great revolutionary development which has given mankind the ability to state general goals and to lay out a path of detailed decisions to take in order to 'best' achieve its goals when faced with practical situations of great complexity." (George Dantzig)

Why is the Module named Exploring Patterns in Linear and Quadratic Relationships?

The ability to look for and make use of structure is important in the study of functions. In this course, students transition from the simpler functions of earlier courses—linear, quadratic, and exponential—to the complexity of polynomial, rational, radical, and logarithmic, functions. Knowing the key and defining characteristics of each function type helps students to build a systematic understanding of algebraic functions and their relationships. This module broadens students' experiences with systems of equations. They learn strategies to solve more complex systems that can model more complex real-world phenomena. Students recall the structure of quadratic functions degree-2 polynomials—a familiar function type from previous courses.

What is the mathematics of Exploring Patterns in Linear and Quadratic Relationships?

Exploring Patterns in Linear and Quadratic Relationships contains three topics: *Extending Linear Relationships, Exploring and Analyzing Patterns,* and *Applications of Quadratics*. The module extends what students know about linear functions by introducing new strategies for solving systems of equations, graphing and solving inequalities, and absolute value functions. The topics in this module extend students' familiarity with linear relationships to derive absolute value functions and to solve problems involving linear systems with more than two equations and two variables.

To begin *Extending Linear Relationships*, students review what they have learned about solving a system comprising two equations and two variables by solving graphically and algebraically. Systems are extended to include three equations and three variables, and students learn to use Gaussian elimination to find solutions. They review systems of linear inequalities and use linear programming to determine optimal solutions to a real-world problem. Finally, students explore matrices.

Then, students are reminded of what they know about the absolute value of a number. A number and its opposite are reflections across $x = 0$ and therefore, have the same absolute value. Students build from this to understand

that taking the absolute value of a function reflects the negative y-values across the x-axis, or the line $y = 0$. The visual representation of an absolute value function allows students to see that there are two x-values associated with each y-value. From this intuitive understanding, students combine what they know about solving equations and inequalities to solve absolute value equations and inequalities.

Exploring and Analyzing Patterns focuses on the different representations of functions. Students begin the topic exploring three different patterns that are modeled by functions from three different function families. After describing and extending the patterns, students represent them numerically, graphically, and algebraically. They determine whether different expressions representing each function are equivalent both graphically and algebraically. Students then dive deep into the structure of degree-2 polynomials. They write quadratic equations given two or three points, and they solve quadratic equations using Properties of Equality, factoring, completing the square, and the Quadratic Formula. Students explore the complex number system, operate with complex numbers, and learn to solve quadratics equations with complex solutions.

In *Applications of Quadratics*, students apply what they know about inequalities, systems, and regressions to the quadratic function family. Throughout the topic, students solve problems in context that require a combination of these skills and make sense of their solutions in terms of the problem situation.

How is Exploring Patterns in Linear and Quadratic Relationships connected to prior learning?

Students use what they know about the absolute value of numbers to understand and solve absolute value equations.

Students have had extensive experience solving systems of equations graphically and algebraically, and their understanding of the structure of equations now prepares them to model more complex situations that require more than two equations and two variables. Throughout their study of algebra, students are expected to construct, solve, and graph equations to represent relationships between two quantities. This module provides students with opportunities to develop strategies focused mainly on linear equations, functions derived from linear relationships, and quadratic functions. As they continue on their mathematical journeys, they will encounter many nonlinear functions, including degree-3 polynomials, rational, and radical functions.

When will students use the knowledge from Exploring Patterns in Linear and Quadratic Relationships in future learning?

Students will spend the remainder of this course studying more complex nonlinear functions. They will use what they know about linear functions, linear systems, and functions derived from linear relationships to generalize to all function types. Students will use what they know about multiple representations of functions and the structure of functions and their key characteristics to expand the inventory of functions that they know and with which they can model scenarios. They will analyze each type of function that follows—rational, radical, exponential, and logarithmic—using the same representations and strategies that they are learning in this module. The importance of analyzing the structure of the function is invaluable to this later work with complex function types.

Students who choose to pursue careers in engineering, economics, statistics, and mathematics will model complex scenarios in systems of equations and will learn to program machines to perform the computations.