TEXAS MATH SOLUTION

Algebra 2

Student Edition

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LONG + LIVE + MATH

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Mathematics is so much more than memorizing rules. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing(TM)—you need to actively engage with the content if you are to benefit from it. The lessons were designed to take you from your intuitive understanding of the world and build on your prior experiences to then learn new concepts. My hope is that these instructional materials help you build a deep understanding of math.

Sandy Bartle Finocchi, Chief Mathematics Officer

You have been learning math for a very long time—both in school and in your interactions in the world. You know a lot of math! In this course, there's nothing brand new. It all builds on what you already know. So, as you approach each activity, use all of your knowledge to solve problems, to ask questions, to fix mistakes, and to think creatively.

Amy Jones Lewis, Director of Instructional Design

At Carnegie Learning we have created an organization whose mission and culture is defined by your success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in you. Our hope is that you will enjoy our resources as much as we enjoyed creating them.

Barry Malkin, CEO, Carnegie Learning

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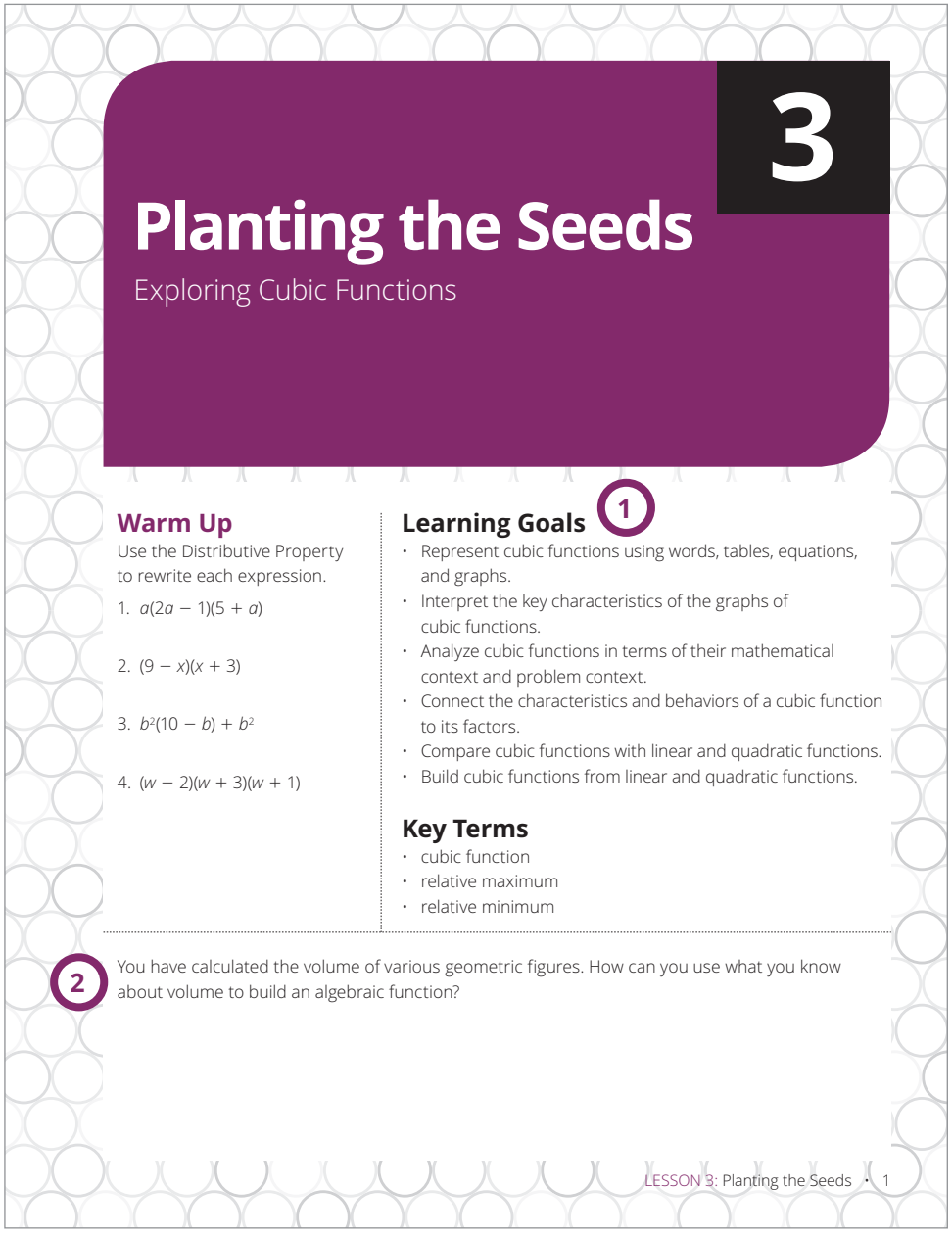
LESSON STRUCTURE

Each lesson has the same structure. Key features are noted.



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Planting the Seeds

Exploring Cubic Functions

Warm Up

Use the Distributive Property to rewrite each expression.

1. $a(2a - 1)(5 + a)$
2. $(9 - x)(x + 3)$
3. $b^2(10 - b) + b^2$
4. $(w - 2)(w + 3)(w + 1)$

Learning Goals

1

- Represent cubic functions using words, tables, equations, and graphs.
- Interpret the key characteristics of the graphs of cubic functions.
- Analyze cubic functions in terms of their mathematical context and problem context.
- Connect the characteristics and behaviors of a cubic function to its factors.
- Compare cubic functions with linear and quadratic functions.
- Build cubic functions from linear and quadratic functions.

Key Terms

- cubic function
- relative maximum
- relative minimum

2 You have calculated the volume of various geometric figures. How can you use what you know about volume to build an algebraic function?

LESSON 3: Planting the Seeds • 1

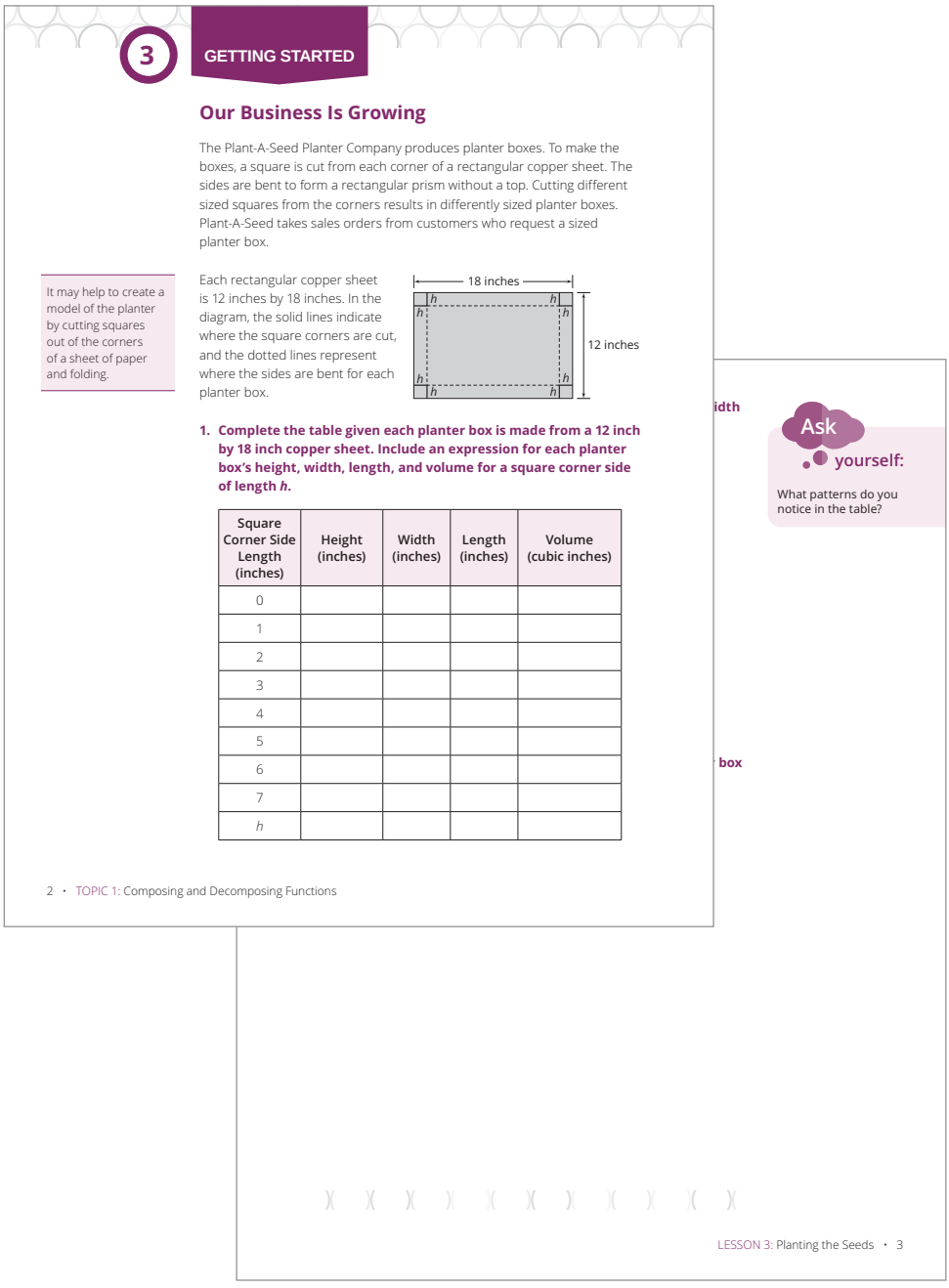
1. Learning Goals
Learning goals are stated for each lesson to help you take ownership of the learning objectives.

2. Connection
Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

3. Getting Started

Each lesson begins with Getting Started. When working on Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.



3 GETTING STARTED

Our Business Is Growing

The Plant-A-Seed Planter Company produces planter boxes. To make the boxes, a square is cut from each corner of a rectangular copper sheet. The sides are bent to form a rectangular prism without a top. Cutting different sized squares from the corners results in differently sized planter boxes. Plant-A-Seed takes sales orders from customers who request a sized planter box.

It may help to create a model of the planter by cutting squares out of the corners of a sheet of paper and folding.

Each rectangular copper sheet is 12 inches by 18 inches. In the diagram, the solid lines indicate where the square corners are cut, and the dotted lines represent where the sides are bent for each planter box.

1. Complete the table given each planter box is made from a 12 inch by 18 inch copper sheet. Include an expression for each planter box's height, width, length, and volume for a square corner side of length *h*.

Square Corner Side Length (inches)	Height (inches)	Width (inches)	Length (inches)	Volume (cubic inches)
0				
1				
2				
3				
4				
5				
6				
7				
h				

2 • TOPIC 1: Composing and Decomposing Functions

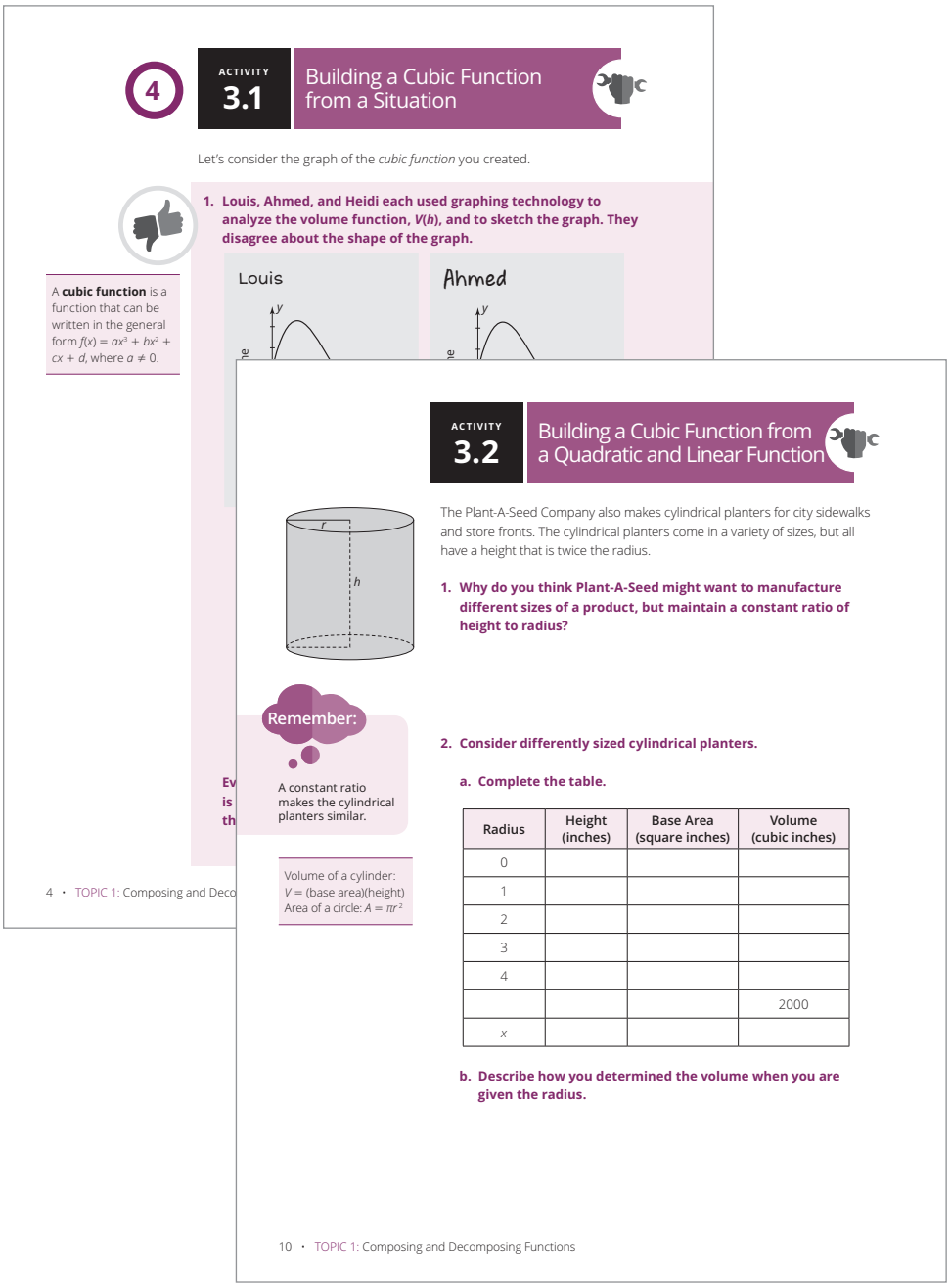
idth

box

Ask yourself:

What patterns do you notice in the table?

LESSON 3: Planting the Seeds • 3



4

ACTIVITY 3.1

Building a Cubic Function from a Situation

Let's consider the graph of the *cubic function* you created.

1. **Louis, Ahmed, and Heidi each used graphing technology to analyze the volume function, $V(h)$, and to sketch the graph. They disagree about the shape of the graph.**

Louis

Ahmed

A **cubic function** is a function that can be written in the general form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

4 • TOPIC 1: Composing and Deco

ACTIVITY 3.2

Building a Cubic Function from a Quadratic and Linear Function

The Plant-A-Seed Company also makes cylindrical planters for city sidewalks and store fronts. The cylindrical planters come in a variety of sizes, but all have a height that is twice the radius.

1. **Why do you think Plant-A-Seed might want to manufacture different sizes of a product, but maintain a constant ratio of height to radius?**

Remember:

A constant ratio makes the cylindrical planters similar.

2. **Consider differently sized cylindrical planters.**

   a. **Complete the table.**

Radius	Height (inches)	Base Area (square inches)	Volume (cubic inches)
0			
1			
2			
3			
4			
			2000
x			

   b. **Describe how you determined the volume when you are given the radius.**

Volume of a cylinder: V = (base area)(height)
Area of a circle: $A = \pi r^2$

10 • TOPIC 1: Composing and Decomposing Functions

4. Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, worked examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

5. Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

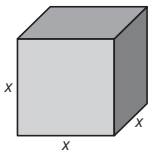
Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

NOTES

5 TALK the TALK

Cubism

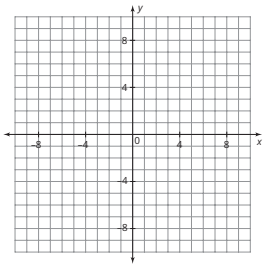
Consider a cube, which has equal length, width, and height, *x*.



1. **Recall that one way to determine the volume of a cube is to multiply the area of the base by its height.**

   a. **Sketch a graph of the function that represents the area of the base of the cube.**

   b. **Sketch a graph of the function that represents the height of the cube.**



   c. **Sketch a graph of the function that represents the volume of the cube.**

2. **Which general shape does this cubic function match? Explain your reasoning.**

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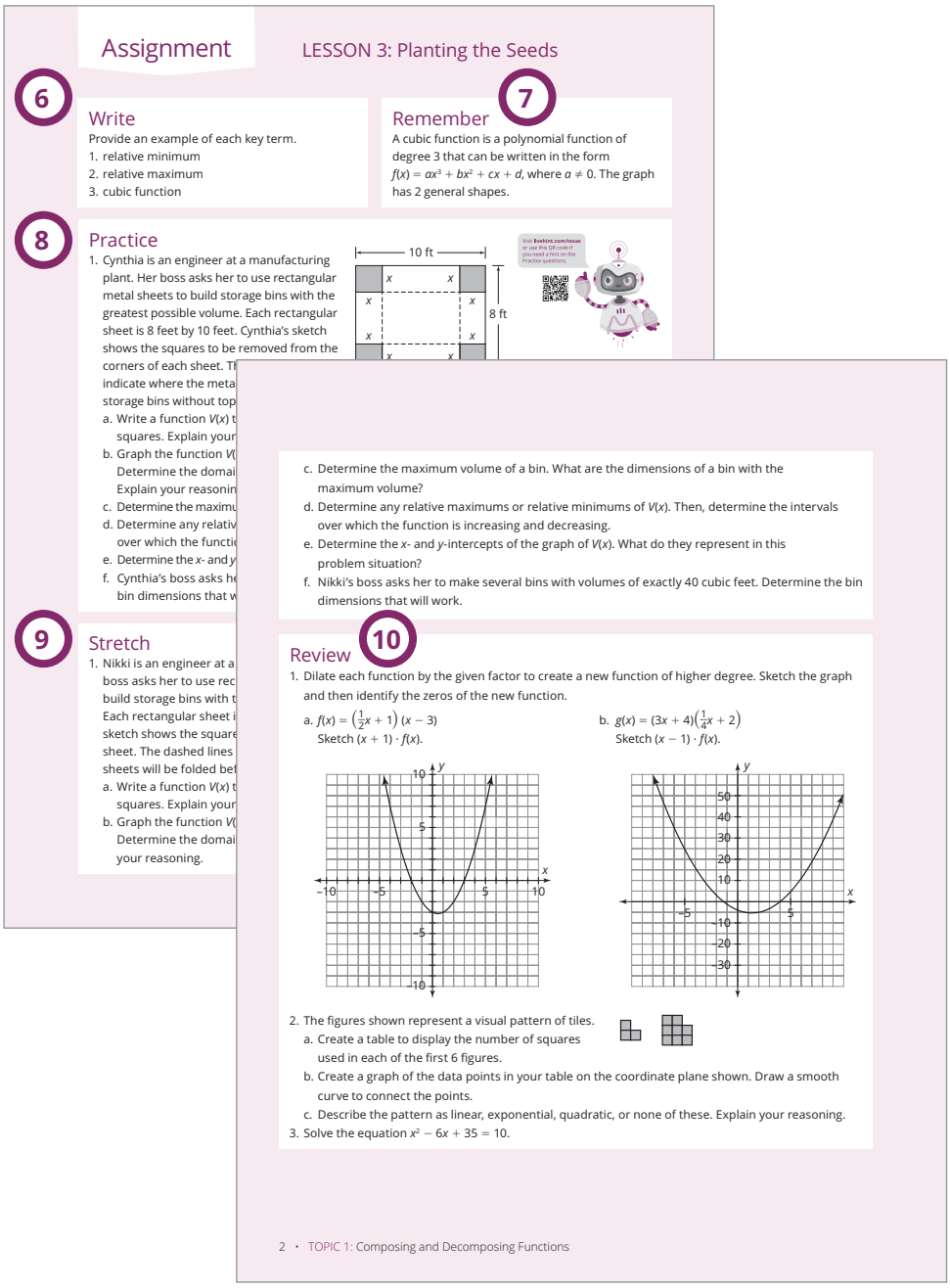
ASSIGNMENT



Assignment

LESSON 3: Planting the Seeds

6

Write

Provide an example of each key term.

1. relative minimum
2. relative maximum
3. cubic function

7

Remember

A cubic function is a polynomial function of degree 3 that can be written in the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$. The graph has 2 general shapes.

8

Practice

1. Cynthia is an engineer at a manufacturing plant. Her boss asks her to use rectangular metal sheets to build storage bins with the greatest possible volume. Each rectangular sheet is 8 feet by 10 feet. Cynthia's sketch shows the squares to be removed from the corners of each sheet. Th
indicate where the meta
storage bins without top
a. Write a function $V(x)$ t
squares. Explain your
b. Graph the function $V($
Determine the domai
Explain your reasonin
c. Determine the maximu
d. Determine any relativ
over which the functi
e. Determine the x- and y
f. Cynthia's boss asks he
bin dimensions that w

10 ft

8 ft

9

Stretch

1. Nikki is an engineer at a
boss asks her to use rec
build storage bins with t
Each rectangular sheet i
sketch shows the square
sheet. The dashed lines
sheets will be folded bet
a. Write a function $V(x)$ t
squares. Explain your
b. Graph the function $V($
Determine the domai
your reasoning.

c. Determine the maximum volume of a bin. What are the dimensions of a bin with the maximum volume?
d. Determine any relative maximums or relative minimums of $V(x)$. Then, determine the intervals over which the function is increasing and decreasing.
e. Determine the x- and y-intercepts of the graph of $V(x)$. What do they represent in this problem situation?
f. Nikki's boss asks her to make several bins with volumes of exactly 40 cubic feet. Determine the bin dimensions that will work.

10

Review

1. Dilate each function by the given factor to create a new function of higher degree. Sketch the graph and then identify the zeros of the new function.

a. $f(x) = \left(\frac{1}{2}x + 1\right)(x - 3)$
Sketch $(x + 1) \cdot f(x)$.

b. $g(x) = (3x + 4)\left(\frac{1}{4}x + 2\right)$
Sketch $(x - 1) \cdot f(x)$.

2. The figures shown represent a visual pattern of tiles.
a. Create a table to display the number of squares used in each of the first 6 figures.
b. Create a graph of the data points in your table on the coordinate plane shown. Draw a smooth curve to connect the points.
c. Describe the pattern as linear, exponential, quadratic, or none of these. Explain your reasoning.
3. Solve the equation $x^2 - 6x + 35 = 10$.

2 • TOPIC 1: Composing and Decomposing Functions

6. Write
Reflect on your work and clarify your thinking.

7. Remember
Take note of the key concepts from the lesson.

8. Practice
Use the concepts learned in the lesson to solve problems.

9. Stretch
Ready for a challenge?

10. Review
Remember what you've learned by practicing concepts from previous lessons and topics.

PROBLEM TYPES YOU WILL SEE

Worked Example

When you see a Worked Example:

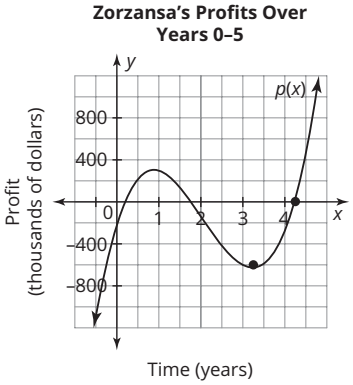
- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

Worked Example

You can determine the average rate of change of Zorzansa's profit for the time interval (3.25, 4.25).



Substitute the input and output values into the average rate of change formula.

Evaluate the expression.

$$\frac{f(b) - f(a)}{b - a} = \frac{f(4.25) - f(3.25)}{4.25 - 3.25}$$

$$= \frac{0 - (-600)}{1}$$

$$= \frac{600}{1} = 600$$

The average rate of change for the time interval (3.25, 4.25) is approximately \$600,000 per year.

Who's Correct

When you see a Who's Correct icon:

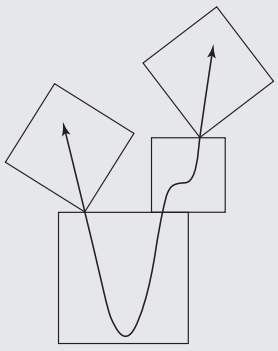
- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine correct or not correct.

Ask Yourself:

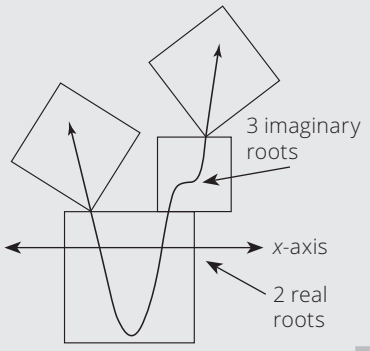
- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

4. Novena created this graph of a fourth degree polynomial. Armondo said that she is incorrect, that it is a fifth degree polynomial. Who is correct? For the student who is incorrect, explain the error in their thinking.

Novena



Armondo



Thumbs Up

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

Ask Yourself:

- Why is this method correct?
- Have I used this method before?

Augie

The cubic function $f(x) = (x - 3)(x - 1)(x + 4)$ has the three zeros given. I can verify this by solving the equations $x - 3 = 0$, $x - 1 = 0$, and $x + 4 = 0$.

Thumbs Down

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

Emily

A cubic function must have three zeros. I know this from the Fundamental Theorem of Algebra. However, the number of real and imaginary zeros can vary. The function may have 0, 1, 2, or 3 imaginary zeros.

MATHEMATICAL PROCESS STANDARDS

Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I can" expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

▶ Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

▶ Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem solving process and reasonableness of the solution.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

▶ Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

▶ Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

▶ Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

▶ Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

▶ Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

ACADEMIC GLOSSARY

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

Visit the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.

ANALYZE

Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

EXPLAIN YOUR REASONING

Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

Related Phrases

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

REPRESENT

Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

Related Phrases

- Predict
- Approximate
- Expect
- About how much?

ESTIMATE

Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

DESCRIBE

Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

Thought Bubbles

Look for these icons as you journey through the textbook. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are here to help and guide your learning.

Side notes are included to provide helpful insights as you work.